Informing Adaptation under Booms and Busts

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Abstract

Leading employees to adapt to changing market conditions – e.g., increased competition or technological change – is an important concern for managers in many business situations. Insufficient adaptation may arise either because employees do not have the information necessary to adapt or because they are not sufficiently incentivized. This paper considers a contracting problem in which a principal is privately informed of market conditions and wants to induce an agent to choose the adaptive project that best fits these conditions. The contract plays a dual role in both motivating and informing the agent. Here, the principal faces a trade-off: A contract that reveals the market conditions fosters adaptation, but it may demotivate the agent. In a dynamic setting, the equilibrium contracts never fully reveal the market conditions, ultimately resulting in insufficient adaptation. My model predicts that adaptation will be inefficiently delayed in booms but happen early in busts. Both cases exhibit organizational inertia and path-dependence: There is a tendency toward continuing old projects.

JEL Classifications: D23, J41, M52

Key words: adaptation, informed principal, incentive contract, commitment, inertia

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1 Introduction

How do firms adapt to changing market conditions when managers are more informed than employees about these changes? In many business situations, managers may possess superior information over employees about market changes as well as the impact of these changes on firms. They are likely to be more capable of analyzing the business environment and in a more advantageous position to acquire related information.[1] On the one hand, firms need a visionary leader to inform employees to make adaptive changes. On the other hand, privately informed leaders may hide or even manipulate the information when doing so is profitable. For example, empirical studies (Kedia and Philippon, 2009, Povel, Singh, and Winton, 2007) suggest that managers may purposely send wrong messages to employees by over-hiring and making excessive investment.

This paper studies firms' adaptation dynamics when managers possess superior information about market conditions over employees. It focuses on the potential consequences of information asymmetry in distorting organizational adaptation behaviors. I construct a dynamic signalling framework that studies how a privately informed manager (the principal, she) designs the employee's (the agent, he) contract to inform and motivate adaptive changes. In my model, only the principal knows how the firm's productivity is hit by market changes over time and only the agent could implement adaptive changes. It thus incorporates both signalling and moral hazard problems into the contract design. The paper analyzes equilibirum adaptation paths and the underlying incentive systems that support these paths. It offers new insights on the adverse role of information asymmetry in resulting in adaptation failures. More interestingly, it predicts the economic conditions whereby adaptation takes place either early or late.

The basic model consists of two periods and two players - a principal and an agent. The market condition can be either good or bad and changes over time in a persistent way; being in a good market today increases the likelihood of being in a good market tomorrow. The principal privately knows the initial market condition and, in the second period, learns about any changes. In each period, the agent implements one of two strategies - a low-cost strategy or a high-cost strategy. The low-cost strategy has a fixed success probability that is known to all. The high-cost strategy is more costly for the agent to implement but has a potentially higher probability of success if

[1]For example, Quigley (1993) conducted a survey of managers in 20 countries, 95% of whom reported that the most important CEO trait is the ability to convey a strong sense of vision to employees. Board directors have experiences and connections in multiple industries (Casal and Caspar, 2014, Larcker, Saslow, and Tayan, 2014), and the top management team could access confidential client and market data that is not accessible to employees.

the market condition is good. The high-cost strategy adapts to a good market due to its high productivity under a good market, and the low-cost strategy adapts to a bad market due to its low cost of implementation.

Adaptation is defined as choosing the strategy that best fits the current market condition. The principal faces the following trade-off: A contract that reveals the market conditions to the agent fosters adaptation, but it may demotivate the agent. The principal, who is privately informed of the worsening prospects of the existing strategy, does not wish to reveal this bad news, as doing so demotivates the agent and increases the incentive cost of retaining her. Anticipating that the principal may conceal bad news, the agent grows suspicious of the principal's true vision and the efficacy of an adaptive strategy. The principal with good news, therefore, has to offer a high salary to induce the agent to implement an adaptive strategy.

Information asymmetry is key to understanding insufficient adaptation as a result of incentive problems. If the principal and the agent are equally informed of the changing market, I show that the agent implements the strategy that adapts to current market conditions. In contrast, under information asymmetry, the equilibrium contracts exhibit insufficient adaptation. By hiding a bad market condition from the agent, the principal in a bad market condition can reduce the incentive cost. The principal that truly operates in a good market therefore has to incur a cost of information revelation (a salary which has zero incentive value) to convince the agent of the good news; not doing so would increase the principal's incentive cost due to the agent's suspicion. If the information revelation cost outweighs the incentive cost, the principal with good news chooses not to reveal the market condition, leading to insufficient adaptation.

This basic trade-off creates two interesting adaptation paths in a dynamic setting; each is unique for a given set of parameter values. In one equilibrium, which I call *early adaptation*, the principal with good news reveals the market condition in the first period, and does so in the second period but only after an initial success. In the other equilibrium, which I call *late adaptation*, the principal with good news only reveals the market condition in the second period, and does so in the second period but only after an initial failure.

The intuition for the two equilibrium paths can be briefly explained as follows. In the early-adaptation equilibrium, a principal with good first-period news commits to a high-powered incentive plan that leaves rents to the agent in the second period after a first-period failure. Such commitment reduces the information revelation cost in the first period. This is because a principal with bad news is more likely to have bad performance in the first period and thus finds it more costly to make the same commitment. On the contrary, this commitment device is not useful following a first-period success, as the principal with first-period good news is more likely to achieve good performance.

In the late-adaptation equilibrium, a principal with good news does not reveal her private information in the first period. The agent therefore makes a negative inference about the market condition after an initial failure. This inference works to the disadvantage of the principal with good first-period news. Information revelation following an initial failure therefore saves the principal from a high incentive cost that she would otherwise have to pay due to the negative inference. This equilibrium features insufficient adaptation (resp., high incentive pay) in the first period and in the second period following a first-period success but adaptation (resp., high salary) in the second period following a first-period failure. The result that negative performance shocks can lead to greater information sharing and foster adaptation is also reminiscent of the Schumpeterian view that economic downturns could play a positive role in promoting long-run growth.

Comparing the two adaptation paths, early adaptation arises if market conditions are sufficiently volatile (good news today is weakly correlated with good news tomorrow). In this case, the agent believes that there is a very high probability of being in a bad market in the second period. The incentive cost of not revealing a good market condition in the first period would outweigh the information revelation cost. The principal with good news therefore reveals the market condition early in the first period. If market conditions are sufficiently stable (good news today is strongly correlated with good news tomorrow), the principal in a good market condition can pool with the principal in a bad market condition at a lower incentive cost in the second period. Intuitively, a stable market outlook reduces the incentive cost for a principal with good news relative to the information revelation cost. These results are also consistent with the finding of Povel, Singh, and Winton (2007) that fraud is most likely to occur in relatively good times and get revealed in the bust.

This paper derives a rich set of dynamic implications of hiding market conditions from employees on distorting adaptation incentives and creating inefficient technology adoption and compensation structure. A volatile market encourages early adaptation but leads to commitment problems that prevent future adaptation after an initial failure. This equilibrium features adaptation (resp., high fixed pay) in the first period and in the second period following a first-period success but insufficient adaptation (resp., high performance-based pay) in the second period following a first-period failure. This is because the agent would resist any reconfiguration of the high-powered compensation plan if the principal wants to save incentive cost from further information revelation in the second period after an initial failure. A stable market, on the contrary, leads to late adaptation in which an initial failure encourages future adaptation. This equilibrium features insufficient adaptation (resp., excessive performance -based pay) in the first period and in the second period following a first-period success but adaptation (resp., high fixed pay) in the second period following a first-period failure. In addition, my paper shows that both the equilibrium paths exhibit organizational inertia and path-dependence: The agent continues to implement the old strategy that no longer fits a new market condition.

In thinking about adaptation paths and compensation structures, it is useful to have examples. What happened to the U.S. auto industry during the Great Depression, a period with volatile market outlook[2], illustrates the early adaptation. As the founder of Chrysler Corporation, Walter Chrysler made the decision to accelerate out of the crisis by investing heavily in research and development and to merge with Dodge in order to take advantage of its strong dealer network. To incentivise the employees to make adaptive changes and learn new technologies, Chrysler paid wage rates higher than the industry average and even increased its wage in 1930s.[3] Many of their innovations quickly became industry standard.[4] As oil prices began to surge in the 1970s, inertia however started to bite the company after it reported a $4 million loss in 1969. Its failure in adapting to the new market condition was partly down to the difficulty of renegotiating a new contract with its employees. The U.S. government eventually had to bail it out with a complex rescue package that left it with the same union contracts, pension obligations, and health care coverage.[5]

In contrast, the development of the internet industry in 1990s, a period with stable economic growth, is an example of late adaptation. Internet companies, including numerous imitators, emerged with ferocity and frequency following the first release of the Mosaic web browser in 1990s. Employees were frequently misled regarding alleged positive prospects of their company. One example is InfoSpace. At its height, the firm was worth $31 billion. Once the company deteriorated, an investigation was carried out, which discovered that revenue came from nonexistent transactions and accounting tricks. By replacing cash-based salaries with stock options, executives deceived their employees by creating a false sense of wealth that was predicated on the slim possibility that the company would succeed. Only after the bursting of the internet bubble were the industry leaders able to stand out and consolidate the market.

[2]For further details, please refer to https://www.bcgperspectives.com.

[3]The automobile industry, which, itself, was one of the highest wage-rate industries in the country. For further details, please refer to https://www.chryslerclub.org/walterp.html.

[4]For example, Chrysler was the first manufacturer to use wind-tunnel testing as part of a design and engineering process that produced more aerodynamically efficient cars.

[5]For further details, please refer to http://ritholtz.com/2008/11/looking-at-the-1980-chrysler-bailout.

Finally, I discuss how renegotiation affects the equilibrium and policies that facilitate adaptive changes. I find that the result of adaptation inertia is robust to renegotiation led by the principal but only survives under certain parameter ranges if it is led by the agent. In terms of policy implications, my model suggests that the government can either subsidize good performers or tax bad performers based on their after-compensation earnings. Such policies create differential effects on firms in different markets: they reward good performers in the better market condition more and penalize bad performers in the worse market condition more, which reduces the information revelation cost and encourages full adaptation.

Related literature. The paper relates to a number of different literatures. At a formal level, I analyze a two-armed bandit problem (Manso, 2011) under a dynamic signalling framework adapted from static signalling models of Fuchs (2015), Maskin and Tirole (1992), Myerson (1983), Zábojník (2014). The model also sheds light on a range of topics including earning management, compensation structure, organizational structure, human capital investment and managerial over-confidence.

(1) Informed principal models. The model is intellectually indebted to the literature on informed principal models (Maskin and Tirole, 1992, Myerson, 1983). The general intuition of their models is: When a party designing a contract has private information, the structure of the contract may reveal some of what he knows to other parties. Compared to Maskin and Tirole (1992), Myerson (1983) considers a more general setup in which agents also have private information and can make private decisions. My paper extends their basic setup to a dynamic one and finds new implications for contract designing. My paper also speaks to the managerial compensation literature that applies informed principal models. For example, Fuchs (2015) studies base salary as a signalling device, but the author leaves the discussion of incentive pay aside. Zábojník (2014) explores the incentive effect of subjective pay but restricts attention to the separating equilibrium. In contrast, I examine both seperating and pooling equilibrium and explore the trade-off between the incentive cost and information revelation cost. Their papers also only consider an informed principal of a constant type.

(2) Incentive contracts and innovation projects. The construction of project choices in my paper is adapted from the literature on incentive contracts and innovation (Holmström, 1989, Manso, 2011). Holmström (1989) explains why small firms are responsible for a disproportionate share of innovative research. Mixing hard-to-measure activities (innovation) with easy-to-measure activities (routine) is more costly for larger firms with a heterogeneous set of tasks. Manso (2011) characterizes the optimal contract that motivates an employee to conduct either exploration-based or exploitation-based innovation under the setting that neither the employee nor the firm knows the exact success probability of the exploration-based innovation. My paper departs from this literature by focusing on information asymmetry in market conditions. In particular, the principal knows the underlying market condition that affects the success probability of the high-cost strategy. The principal in my model is concerned with transmitting information from the top to the bottom rather than learning the quality of the exploratory project.

(3) Earnings management. The paper is close to the earning management literature. Prior theoretical work (see Narayanan (1985), Stein (1989), Goldman and Slezak (2006) and Povel, Singh, and Winton (2007)) study how informed managers divert valuable resources to misrepresent performance at the cost of uninformed investors or shareholders. Kedia and Philippon (2009) investigate of the real effects of earnings management on the hiring and investment. Specifically, they find that during the misreported period, firms hire and invest more than comparable firms, while hiring and investment are significantly lower after the restated period. My paper focuses on hiding negative information from uninformed employees and offers new predictions on its dynamic effects on employees' compensation structures and adaptation choices.

(4) Organizational structure and adaptation. Dessein and Santos (2006) and Rantakari (2008) emphasize the importance of coordination and authority in influencing adaptation. In their setting, divisional managers (the agent) rather than the headquarters (the principal) have direct access to information about local market conditions. A fundamental difference between this paper and other explanations lies in the premise of the origin of adaptation. While other papers consider adaptive changes to be initiated by specialized employees and divisional managers, as in Mintzberg and Waters (1985), this paper takes the view of Bennis and Nanus (1985) and Quigley (1993) that visionary senior managers are the primary driving force of adaptation in organizations.[6,7]

Relatedly, Dow and Perotti (2010) posits that firms fail to adapt to changed circumstance because losers from the radical adjustment can credibly resist and oppose changes, which explains the creation of ambidextrous firms. Both the firm and agent understand the market changes in their setting. Critically, that paper assumes that

[6]For example, CEOs could be hired for their vision in exploiting new potentials dormant in the market. Quigley (1993) conducted a survey of managers in 20 countries, 95% of whom reported that the most important CEO trait is the ability to convey a strong sense of vision to employees. Board directors have experiences and connections in multiple industries (Casal and Caspar, 2014, Larcker, Saslow, and Tayan, 2014), and the top management team could access confidential client and market data that is not accessible to employees.

[7]Effective communication of changing market conditions to employees is doubtless a key component of successful adaptation (Covin and Kilmann, 1990, Lewis, 2006).

output is not verifiable and therefore remains silent on the issues of incentive contracts.

(5) Project-specific investment in human capital. Another related literature on commitment and strategy specific investment also helps explain the difficulty of adopting adaptive strategies. Rotemberg and Saloner (1994) and Van den Steen (2005) show that managerial vision, or a bias towards a specific strategy, provides employees with more certainty that their strategy-specific investments will pay off. Mailath, Nocke, and Postlewaite (2004) consider mergers and argue that merging, by making unprofitable certain decisions, increases the cost of inducing managerial effort. Ferreira and Rezende (2007) show that reputation-concerned managers could use the public disclosure of strategic plans as a mechanism of commitment to a specific strategy. Bolton, Brunnermeier, and Veldkamp (2013) argue that a CEO's overconfidence regarding the quality of his initial information on the firm's optimal strategy serves as a commitment device. Those models posit that firms optimally maintain a narrower set of strategies to resolve the time-inconsistency problem and to encourage employees to invest in task-specific skills. My paper focuses on the adverse role of the informational friction in creating incentive and commitment problems that lead to long-run adaption failure.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the benchmark case. Second 4 analyses the equilibrium adaptation paths. Section 5 presents the equilibrium compensation structure that supports each adaptation path. Section 6 considers several extensions and discusses empirical predictions. The last section concludes.

2 Model Setup

The model consists of two periods (period $t = 1$ and 2). There are two players, a principal and an agent.

2.1 Dynamic Environments

The market condition m_t, in each period, can be in one of two possible states, $m_t \in \{G, B\}$. G (resp., B) represents a good (resp., bad) market condition.

At date 0, the prior probabilities of m_1 being G and B are α and $1-\alpha$, respectively, $0 < \alpha < 1$. The market condition might change in the second period. With probability β, a good market condition continues to be good, $Pr(m_2 = G|m_1 = G) = \beta$, and $0 < \beta < 1$. With probability $1 - \beta$, a good market condition deteriorates, $Pr(m_2 = B|m_1 = G) = 1 - \beta$. Parameters α and β are known to both parties.

A bad market condition in the first period remains bad in the second period, $Pr(m_2 = B|m_1 = B) = 1$. This assumption greatly simplifies the analysis by reducing the number of states of economy. The main results of the paper is robust to this simplification. As long as the market change is persistent, the result of path-dependency of adaptation holds in a more general setting in which a bad market could improve in the second period. This assumption also provides a robust setting to study excessive adoption of the high-cost strategy, as it is least likely to occur under this assumption.

To summarize, the market changes persistently - a good market today predicts a higher likelihood of a good market tomorrow than a bad market does.

2.2 Two Strategies

The modelling of two strategies is adapted from Manso (2011)'s two-period and two-armed bandit problem. In my model, the agent can choose one of two strategies $s_t \in \{s_h, s_l\}$ in each period t, where s_h is a high-cost strategy and s_l is a low-cost strategy. Implementing either of them yields a verifiable output y_t at the end of each period t, $y_t \in \{0, 1\}$. y_1 and y_2 are independently distributed. The probability of achieving high output, if the agent does not implement any strategy, is zero.

The low-cost strategy generates a high output with a constant probability θ_l in each period t, $Pr(y_t = 1|s_l) = \theta_l$. The high-cost strategy exhibits more uncertainty. Its probability of success θ_t in period t depends on the market condition in that period. For instance, if a newly developed product is well received by consumers, or a firm expands into a foreign market that is undergoing rapid economic growth, then θ_t is high. Otherwise, it is low. To be more specific, θ_t can take two possible values $\theta_t \in \{\theta_l, \theta_h\}$, with $0 < \theta_l < \theta_h \leq 1$.[8] If $m_t = G$, then $\theta_t = \theta_h$, or $Pr(y_t = 1|m_t = G, s_h) = \theta_h$. If $m_t = B$, then $\theta_t = \theta_l$, or $Pr(y_t = 1|m_t = B, s_h) = \theta_l$. Because the market condition may change over time, the high-cost strategy, which has a success probability of θ_h in the first period, may become less productive in the second period.

If the principal and the agent are both uninformed of the underlying market condition, my model resembles the problem solved in Manso (2011) in the sense that the principal and the agent can only learn the market condition by experimenting with the high-cost strategy. Because the market does not affect the output generated by the low-cost strategy, adopting the low-cost strategy does not produce any informational value to the firm.

[8] In a more general setting in which the lower boundary of θ_t is smaller than θ_l in the bad state, the set of equilibrium strategies also inlcudes the low cost strategy under a pooling equilibrium. The result of path-dependency of adaptation still holds in this more general case.

If s_h is implemented, the agent can make an inference from about m_1 from y_1.

$$Pr(m_1 = G|y_1 = 1) = \frac{Pr(y_1 = 1|m_1 = G)Pr(m_1 = G)}{Pr(y_1 = 1)} = \frac{\alpha\theta_h}{\alpha\theta_h + (1-\alpha)\theta_l}$$

The probability of being in a good market in the second period upon observing a high output is therefore:

$$Pr(m_2 = G|y_1 = 1) = \beta Pr(m_1 = G|y_1 = 1) = \frac{\alpha\beta\theta_h}{\alpha\theta_h + (1-\alpha)\theta_l}$$

The probability of achieving high output in the second period given high output in the first period is:

$$\begin{aligned} Pr(y_2 = 1|y_1 = 1) &= \theta_h Pr(m_2 = G|y_1 = 1) + \theta_l(1 - Pr(m_2 = G|y_1 = 1)) \\ &= \frac{\alpha\beta\theta_h(\theta_h - \theta_l)}{\alpha\theta_h + (1-\alpha)\theta_l} + \theta_l \end{aligned}$$

Similarly, the probability of achieving high output in the second period given low output in the first period is:

$$\begin{aligned} Pr(y_2 = 1|y_1 = 0) &= \theta_h Pr(m_2 = G|y_1 = 0) + \theta_l(1 - Pr(m_2 = G|y_1 = 0)) \\ &= \frac{\alpha\beta(1-\theta_h)(\theta_h - \theta_l)}{\alpha(1-\theta_h) + (1-\alpha)(1-\theta_l)} + \theta_l \end{aligned}$$

The unconditional probability of achieving high output in the second period is:

$$Pr(y_2 = 1) = \theta_h Pr(m_2 = G) + \theta_l Pr(m_2 = B) = \alpha\beta(\theta_h - \theta_l) + \theta_l$$

If the principal and the agent are symmetrically uninformed of the market condition, the transition matrix of successes corresponds to those assumed in Manso (2011) and Ferreira, Manso, and Silva (2014). To see this, one could easily verify that $Pr(y_2 = 1|y_1 = 0) < Pr(y_2 = 1) < Pr(y_2 = 1|y_1 = 1)$.[9] Intuitively, if the agent adopts the high-cost strategy, then high output in the first period indicates greater likelihood of a good market condition in $t = 1$ and $t = 2$, and therefore greater probability of high output in $t = 2$. Low output indicates greater likelihood of a bad market condition in $t = 1$ and $t = 2$, and therefore greater probability of low output in $t = 2$. If the market change is not persistent ($\beta = 0$), then $Pr(y_2 = 1|y_1 = 0) = Pr(y_2 = 1) = Pr(y_2 = 1|y_1 = 1) = \theta_l$. In other words, learn-

[9] Manso (2011) and Ferreira, Manso, and Silva (2014) assume that $E(q|F) < E(q) < E(q|S)$. q is the probability of success of the innovative task, which is unknown to both the principal and the agent. F and S mean the success and failure of the task implemented by the agent.

ing from past performance adds additional information to the firm's information set only if the market condition changes persistently. Otherwise, past performance is not indicative of the future market condition.

2.3 Incentive Problems

The principal hires the agent to implement a strategy. The low-cost (resp., high-cost) strategy requires a private effort cost of C_l (resp., C_h) to implement. As in Manso (2011), I assume that the principal does not observe the strategies implemented by the agent.[10] This assumption is meant to capture the difficulty for large organizations in ensuring the implementation of the adaptive strategy due to the separation between the formulation of strategies at the senior manager level and the implementation of strategies at the employee level. As Mintzberg (1988) noted, when implementing strategies, employees usually find themselves between past experiences (or knowledge) and future prospects, and the process of applying past experiences to solving new problems is inherently unobservable. Adding to the difficulty of overcoming the moral hazard problem is a lack of input-based measures, especially in uncertain environments. To ease exposition, I denote $\Delta = C_l - \frac{C_h}{\theta_h}\theta_l$.

Two efficiency conditions hold for the two strategies:

1. **G-efficiency condition**, $\frac{C_h}{\theta_h} < \frac{C_l}{\theta_l} < 1$;
2. **B-efficiency condition**, $C_l < C_h$.

The G-efficiency condition implies that the high-cost strategy is more efficient under a good market condition.[11] The B-efficiency condition implies that the low-cost strategy is more efficient under a bad market condition. I restrict attention to the case in which $C_l < C_h$ and thus rule out situations in which the high-cost strategy dominates the low-cost one in both strict and weak forms.[12] In other words, the high-cost strategy adapts to a good market condition while the low-cost strategy adapts to a bad market condition.[13] Adaptation is defined as the agent implementing the more

[10]The mimicking motive can still be preserved even if the principal observes the agent's project choices, as the principal in the good market condition still faces the truth-telling constraint and needs to make the agent break even.

[11]If $\frac{C_h}{\theta_h} \geq \frac{C_l}{\theta_l}$, the principal in the bad market has no incentive to conceal the bad news, and hence full information revelation automatically arises. To study insufficient information revelation, I focus on the case in which $\frac{C_h}{\theta_h} < \frac{C_l}{\theta_l}$.

[12]If $C_l = C_h$, the high-cost strategy weakly dominates the low-cost strategy. In this case, equilibrium results still hold, but there is no efficiency loss in equilibrium.

[13]Note that the effort cost and success probabilities of these two strategies do not imply that the principal always prefers the high-cost strategy, as the principal has to internalize the incentive cost of motivating the agent to work.

efficient strategy under a certain market condition. I define the concept of adaptation as follows.

Definition *A firm achieves adaptation in t if the agent implements s_h under $m_t = G$ and s_l under $m_t = B$.*

2.4 An Informed Principal

The principal in my model is privately *informed* of m_t in each period t, whereas the principal in Manso (2011) is not. Therefore, learning through experimentation has no informational value to the principal. The agent is uninformed of the market condition throughout. Although she could infer it from past performance by implementing the high-cost strategy, the inference will not be as accurate as the private information that the principal has.

If neither the principal nor the agent is informed of the market condition, then it is more efficient to continue implementing the high-cost strategy following good performance and to return to the low-cost strategy following bad performance. This problem corresponds to the one analysed in Manso (2011) if assuming the same parameter values. In the setup of this paper, continuing to implement the high-cost strategy following good performance, however, may not be efficient, as the principal knows whether the market condition deteriorates or not in the second period.

The economic setting in this paper thus emphasizes situations in which a visionary manager needs to credibly convey her private information to the employee and lead the ill-informed employee to adapt to market changes. Because the principal accurately knows the market condition, the strategy choice made by the agent in the second period depends not only on what she infers from past performance but also on information revealed by the principal. The focus of the contracting problem in this paper is not to encourage experimentation and learning but to facilitate information revelation from the top to the bottom in a hierarchical organization. The ultimate goal of such information revelation is to encourage adaptation. I define the concept of information revelation as follows.

Definition *A firm achieves information revelation in t if the agent shares the same belief of m_t with the principal in t.*

2.5 Preferences

The principal and the agent are both risk neutral. For simplicity, I assume that the discount rate for future payoffs is zero. The principal maximizes the firm's profit,

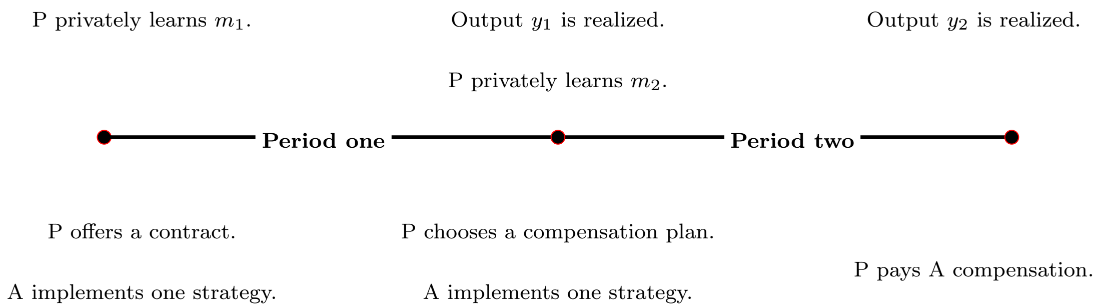


Figure 1: The Time-line

Note: A represents the agent; P represents the principal.

and the agent maximizes her compensation after deducting her effort disutility. She has zero initial wealth and is protected by limited liability. Her reservation utility is assumed to be zero over the entire time horizon, under any market conditions.

2.6 Contracts

Here, I characterize long-term contracts in equilibrium. While performance is contractible, neither the principal's private information nor the agent's strategy choice is.

Based on the private signal m_1, the principal offers the agent a contract $\mathcal{P}_{t=1}$ at the beginning of the first period. The contract is a subset of $\mathbb{R}^4_+$, $\mathcal{P}_{t=1} \subseteq \mathbb{R}^4_+$. Contract $\mathcal{P}_{t=1}$ is a set of compensation plans. I call $p_{t=2}$, an element of $\mathcal{P}_{t=1}$, a compensation plan. $\mathcal{P}_{t=1}$ may contain more than one compensation plan, i.e., $\mathcal{P}_{t=1} = \{p^i_{t=2}, i = 1, 2, ...n\}$, where $p_{t=2} = \{w_{00}, w_{10}, w_{01}, w_{11}\}$. The principal pays the agent w_{00} if $(y_1, y_2) = (0, 0)$, w_{10} if $(y_1, y_2) = (1, 0)$, w_{01} if $(y_1, y_2) = (0, 1)$, and w_{11} if $(y_1, y_2) = (1, 1)$. Limited liability constraints imply that all payments are non-negative. After observing the first-period performance and receiving the private signal m_2 at the beginning of the second period, the principal, at her sole discretion, chooses a single compensation plan $p^i_{t=2}$ from $\mathcal{P}_{t=1}$. If the principal agrees to compensate the agent according to $p^i_{t=2}$, then the agent is paid according to the compensation plan.

For a simple illustration, if the principal limits her choices to one combination of $\{w_{00}, w_{01}\}$ following bad performance ($y_1 = 0$) and two possible combinations of $\{w_{10}, w_{11}\}$ following good performance ($y_1 = 1$), then $\mathcal{P}_{t=1}$ contains two compensation plans $\mathcal{P}_{t=1} = \{p^1_{t=2}, p^2_{t=2}\}$, where $p^1_{t=2}$ and $p^2_{t=2}$ have the same w_{00} and w_{01} but different w_{10} and w_{11}.

Figure 1 presents the timeline. At date 0, the principal is privately informed of the market condition m_1 and offers a contract $\mathcal{P}_{t=1}$ to the agent. The agent could leave or stay. If she leaves, she obtains a reservation utility of zero. If she accepts the contract, she implements one strategy. At the end of the first period, two parties observe the realization of y_1. At the beginning of the second period, after observing the market condition m_2 and y_1, the principal chooses a single compensation plan $p^i_{t=2}$ from the contract $\mathcal{P}_{t=1}$ and offers it to the agent. The agent then implements one strategy again. At the end of the second period, two parties observe the realization of y_2. Compensation is finally paid.

Because the setting involves a signalling problem, the payment scheme will be either fully revealing under a separating Perfect Bayesian Equilibrium (PBE) or not under a pooling PBE. This setting therefore might have multiple equilibria. I use a belief-based refinement approach of Undefeated Equilibrium, introduced by Mailath, Okuno-Fujiwara, and Postlewaite (1993). I will introduce this approach in Section 4 before the analysis of the equilibrium structure of information revelation.

3 Benchmark – Symmetric Information

This benchmark case describes contracts offered by the principal and strategies adopted by the agent when both parties know the market condition at the begining of each period. The timeline of this benchmark case corresponds to the two-period model in Section 2.

Proposition 1 *Assume that both parties know the market condition at the begining of each period.*

- *If $m_1 = G$, then the principal offers a contract $\mathcal{P}_{t=1}$ that consists of two compensation plans $\mathcal{P}_{t=1} = \{p^{GG}_{t=2}, p^{GB}_{t=2}\}$ where $p^{GG}_{t=2} = (w^{GG}_{00}, w^{GG}_{01}, w^{GG}_{10}, w^{GG}_{11})$, and $p^{GB}_{t=2} = (w^{GB}_{00}, w^{GB}_{01}, w^{GB}_{10}, w^{GB}_{11})$.*
  - *For plan $p^{GG}_{t=2}$, $w^{GG}_{01} = \frac{C_h}{\theta_h}$, $w^{GG}_{10} = \frac{C_h}{\theta_h}$, $w^{GG}_{11} = 2\frac{C_h}{\theta_h}$, and $w^{GG}_{00} = 0$;*
  - *For plan $p^{GB}_{t=2}$, $w^{GB}_{01} = \frac{C_l}{\theta_l}$, $w^{GB}_{10} = \frac{C_h}{\theta_h}$, $w^{GB}_{11} = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_h}$, and $w^{GB}_{00} = 0$.*
- *If $m_1 = B$, the principal offers a contract $\mathcal{P}_{t=1} = (w^{BB}_{00}, w^{BB}_{01}, w^{BB}_{10}, w^{BB}_{11})$. Specifically, $w^{BB}_{01} = \frac{C_l}{\theta_l}$, $w^{BB}_{10} = \frac{C_l}{\theta_l}$, $w^{BB}_{11} = 2\frac{C_l}{\theta_l}$, and $w^{BB}_{00} = 0$.*
- *The agent always implements s_h under a good market condition, and s_l under a bad market condition.*

Proposition 1 presents the equilibrium contract of this benchmark case. All proofs are given in the Appendix. Superscripts GB, GG and BB denote the three types of market changes over the two periods. $p^{GG}_{t=2}$, $p^{GB}_{t=2}$, and $p^{BB}_{t=2}$ indicate the compensation plans under the respective market conditions. For example, if the market condition continues to be good in the second period, the principal will choose the compensation plan $p^{GG}_{t=2}$.

The contract under symmetric information exhibits two interesting features. First, the agent is incentivized to implement the strategy that adapts to the current market condition. When the agent also knows the market condition and how it changes over time at the beginning of the second period, the only role of the contract is to motivate the agent to implement the adaptive strategy. In the absence of information asymmetry, firms can achieve full adaptation.

Second, the contract in Proposition 1 requires no commitment because it is sequentially rational to the principal. In fact, as shown in Corollary 1, one could decompose the contract into two short-term contracts, each of which consists of performance-based pay that is contracted only on the current-period performance. That is, the principal in any periods offers a bonus $\frac{C_h}{\theta_h}$ for high output under a good market condition and $\frac{C_l}{\theta_l}$ under a bad market condition, and zero for low output.

Corollary 1 *The contract in Proposition 1 can be implemented by two short-term contracts where w^m_y denotes the pay for output y under a market condition m:*

- *If $m_1 = G$, then the principal offers $w^G_1 = \frac{C_h}{\theta_h}$ and $w^G_0 = 0$ in $t = 1$. In $t = 2$,*
  - *If $m_2 = G$, then the principal continues to offer $w^G_1 = \frac{C_h}{\theta_h}$ and $w^G_0 = 0$;*
  - *If $m_2 = B$, then the principal offers $w^B_1 = \frac{C_l}{\theta_l}$ and $w^B_0 = 0$.*
- *If $m_1 = B$, then the principal offers $w^B_1 = \frac{C_l}{\theta_l}$ and $w^B_0 = 0$ in both periods.*

4 Equilibrium Structure of Information Revelation

This section analyses the equilibrium structure of information revelation under asymmetric information. Given that a number of equilibria can be supported by a variety of off-the-equilibrium beliefs, I use a belief-based refinement approach of Undefeated Equilibrium[14] introduced by Mailath, Okuno-Fujiwara, and Postlewaite (1993). The Intuitive Criterion and D1 (Cho and Kreps, 1987) eliminate all pooling equilibria, among which some are interesting and reasonable. This is due to the lack of a "global"

[14]This refinement approach is also used in several other papers, including Taylor (1999), Gomes (2000), Fishman and Hagerty (2003) and Josephson and Shapiro (2014).

consistency, which neglects all the subsequent adjustments in strategies and beliefs that will take place after a disequilibrium message is sent.[15]

The Undefeated Equilibrium approach selects among different pure strategy PBEs and selects a unique equilibrium outcome for a given set of parameters. In my setting, these equilibria are such that

1. The principal in one type of market uses a pure strategy and maximizes profits given the agent's choices and the pure strategy of the principal in the other type of market in time $t = 1, 2$;

2. The agent chooses either the low-cost or the high-cost strategy conditional on the contract offered by the principal in time $t = 1, 2$;

3. Beliefs in time $t = 1, 2$ are calculated using Bayes' rule for the contract offered by the principal used with positive probability .

Undefeated Equilibrium E is defined as defeating another PBE E' if:

1. There is a message m sent only in E by a set of types K;

2. The set of types K who send m are all better off in E than in E', and at least one of them is strictly so;

3. Off-the-equilibrium beliefs under E' regarding at least one type in K conditional on sending m are not a posterior probability assuming: (i) only types in K send m with positive probability and (ii) those types in K that are strictly better off under E send m with probability one.

A PBE E is said to be undefeated if there does not exist another PBE E' that defeats it. The undefeated approach is essentially a lexicographically maximum refinement concept and works by verifying that no types in one equilibrium are better off in another equilibrium in which they choose a different action/message.

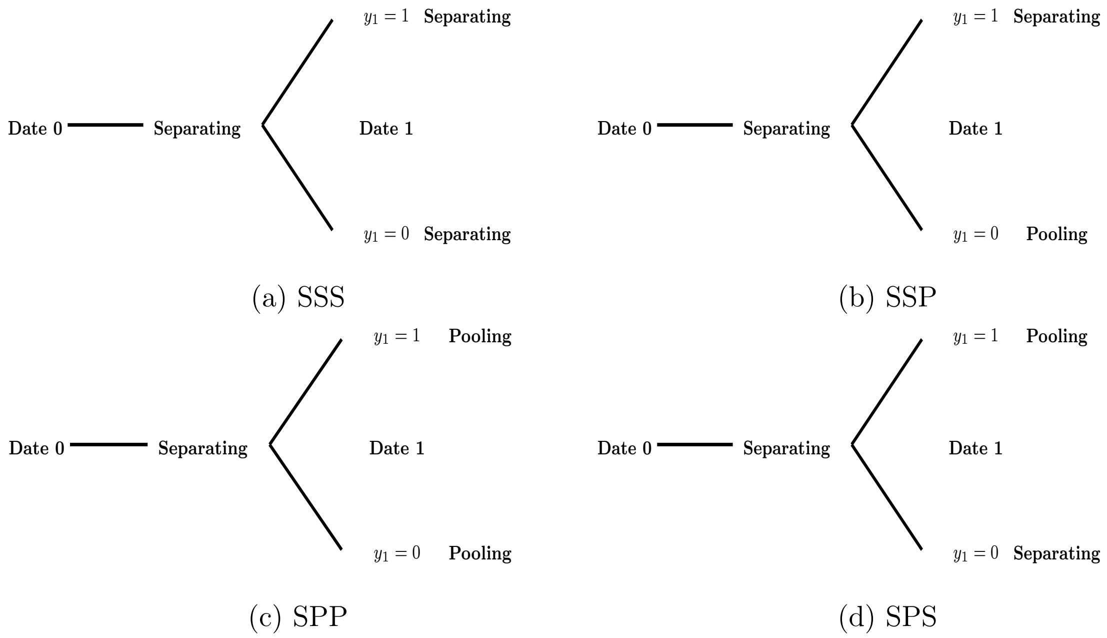
Finally, it might be desirable to place additional requirements[16] on the belief systems of a sequential equilibirum. The agent does not change her belief about the market condition as a result of any deviation of her own. In addition, once the agent

[15] Cho and Kreps attribute this reasoning to Stiglitz. In fact, in a monotonic signaling game or if the sorting condition is satisfied, any equilibrium in which two or more types assign positive probability to the same action must fail the Intuitive Criterion and D1 (Cho and Sobel, 1990). Cho and Kreps also observe that D1 distinguishes the separating equilibrium with three types. A stronger concept of universal divinity proposed by Banks and Sobel (1987) selects the separating equilibrium with more than two types but coincides with D1 when there are only two types. None of them therefore have bite in this model.

[16] These restrictions are highly intuitive and are typically regarded as mild; see, for example, Rubinstein (1985) and Grossman and Perry (1986).

is fully certain about the first-period market condition, her subsequent beliefs of the first-period market condition put positive probability only on that market condition. These constraints imply that the agent does not change her initial belief of the first-period market condition unless the contract chosen in $t = 2$ separates principals across different markets and that she continues to hold the belief of a certain market condition with probability 1 whatever occurs after she comes to this conclusion.

Figure 2 and Figure 3 presents all possible information revelation structures. Specifically, Figure 2 includes those that reveal the market condition in the first period, and Figure 3 includes those that do not. Information revelation structure in the second period depends on the performance realization in the first period.

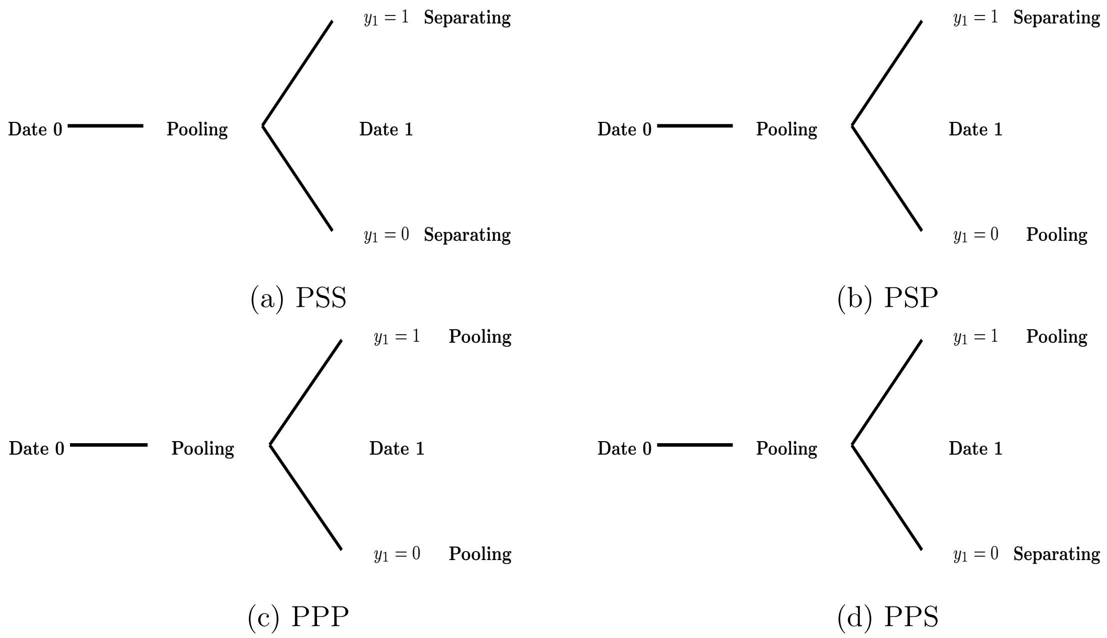


(a) SSS (b) SSP (c) SPP (d) SPS

Figure 2: Possible Structures of Information Revelation I

The analysis of equilibrium contracts involves several complications. First, revealing the bad market condition demotivates the agent, and the principal would have to offer higher performance-based pay to induce the same effort. The principal therefore may not want to reveal the bad news, which gives rise to the tradeoff that the principal in a good market condition has to face. That is, credible revelation of the good news is costly, but it saves some performance-based pay. In other words, at which period and following which performance realization the truth-telling constraint should bind has to be endogenously determined by the principal trading off the two countervailing forces.

Second, one major difference between structures in Figure 2 and those Figure 3 lies



(a) PSS (b) PSP (c) PPP (d) PPS

Figure 3: Possible Structures of Information Revelation II

in how the agent learns the first-period market condition. If the principal does not reveal information in the first period, the agent has to learn it from past performance. If the agent adopts the high-cost strategy, the agent, upon observing good first-period performance, believes that the market is more likely to be good today and tomorrow due to the persistence of market changes. Conversely, upon observing bad first-period performance, the agent believes in a higher likelihood of a bad market condition today and tomorrow. The principal therefore has to consider the sequential inference made by the agent when designing the contract at date 0.

Third, in Figure 2, if there is any further information revelation in the second period, the separation must happen between types GG and GB, as BB is already separated out in the first period. By contrast, as there is no information revelation in the first period in Figure 3, separation in the second period may take three possible forms: separating firms across different markets based on the first- and second-period market conditions (separating types of GG, GB, and BB from each other), or based on the first-period market condition (separating types of BB from (GB, GG)), or based on the second-period market condition (separating types of GG from (GB, BB)).

The following Lemma 1 is important to understand the information revelation structures in Figure 3. It holds that if the principal in the second period reveals any private information, she only reveals the second-period market condition, not the first-period market condition. In other words, if there is information revelation in the second period, a principal in a deteriorating market (GB) and a principal in a constantly bad market (BB) pool together, and a principal in a constantly good market (GG) is separated from them.

Lemma 1 *In any equilibrium in which there is no information revelation in the first period, the principal in the second period only reveals* m_2*, not* m_1*.*

The intuition is very straightforward. If information revelation in the second period concerns only m_1, that is, if principals in GG and GB pool together and separate from principals in BB, the principal in BB would offer the same contract as the principal in GB. This is because the principal in BB in the second period has the same mimicking incentive as the principal in GB. Therefore separation in the second period will not occur between them. In fact, one could easily verify that the second-period incentive constraint and truth-telling constraint for an agent in GB are the same as the ones for an agent in BB. In other words, the first-period private information (m_1) does not enter into the second-period constraints of the maximization programs of principals in GB and BB. Moreover, the first-period information is redundant with the information of the second-period market condition, because the productivity of the agent's strategy choice in the second period is only affected by the second-period market condition once it is revealed. This lemma further indicates that one principal needs at most two compensation plans at the initial contract offering stage.

Before I characterize the equilibrium structures of information revelation, I first describe the strategies implemented by the agent in equilibrium in Lemma 2.

Lemma 2 *In any equilibrium in which the agent does not learn the current market condition, she implements the high-cost strategy. In any equilibrium in which the agent learns the current market condition, she implements the strategy that adapts to the current market condition.*

The first result of Lemma 2 can be intuitively explained as follows. Assume that the equilibrium contract is designed in a way that agents in both markets implement s_l under no information revelation. The principal in a good market is better off offering a contract that reveals the market condition to the agent. This is because the principal would not have to incur a loss in output that is greater than the cost of information revelation.

The second result of Lemma 2 says that if the agent learns of a good market condition, then she implements the high-cost strategy. If the agent learns of a bad market condition, she implements the low-cost strategy. The intuition is simple. Conditional on the good market condition being revealed, if the principal offers the agent a compensation plan that just satisfies the agent's incentive constraint under the low-cost strategy, the agent will always prefer the high-cost strategy because of its high productivity. The principal therefore offers the agent incentive pay that is just sufficient to compensate for the agent's effort of adopting the high-cost strategy. Similarly, conditional on the bad market condition being revealed, if the principal offers the agent a compensation plan that satisfies the agent's incentive constraint under the high-cost strategy, the agent will always prefer the low-cost strategy because of its low cost. The principal therefore offers the agent incentive pay that is just sufficient to compensate for the agent's effort of adopting the low-cost strategy.

Proposition 2 ***The equilibrium information revelation structures.***

- *If α (the probability of $m_1 = G$) and β (the probability of $m_2 = G$ conditional on $m_1 = G$) are sufficiently small, SSP is the equilibrium information structure.*
- *If α and β are sufficiently large, PPS is the equilibrium information structure.*
- *If α is sufficiently small and β is sufficiently large, PSS is the equilibrium information structure.*
- *If α is sufficiently large and β is sufficiently small, PPS is the equilibrium information structure if C_h is sufficiently low, and SSP is the equilibrium information structure if C_h is sufficiently high.*

Proposition 2 characterizes the equilibrium information revelation structures. Figure 2b, Figure 3a and Figure 3d (with captions in blue) graphically represent the equilibrium information revelation structures, each of which is unique given a set of parameter values. I choose to focus on SSP and PPS. These two contracts possess an interesting feature of path-dependency in their information revelation structures – whether information is revealed in the second period depends on performance in the first period. For a full analysis of the three contracts, readers can refer to the Appendix A. I call SSP *early adaptation* and PPS *late adaptation.*[17]

According to Proposition 2, insufficient adaptation is a critical and robust feature in every equilibrium. Full information revelation (SSS), which leads to full adaptation, however, is not an equilibrium. Because revealing bad news concerning the market condition demotivates the agent, the principal in a bad market may not want to do so. The principal in a good market therefore has to incur the cost of information revelation to convince the agent of the good market condition. As I will show in the section on the equilibrium compensation structure, the fully revealing contract is downward rigid.

[17]In fact, the PSS equilibrium also involves late adaptation. To ease the exposition, late adaptation in the paper only refers to PPS.

Because promising a non-decreasing contract is costly, insufficient adaptation arises in equilibrium.

4.1 Early Adaptation

To help illustrate the intuition for early adaptation, I first present the following auxiliary result.

Lemma 3 *If $\alpha < 1/(\theta_h - \theta_l)(\frac{C_h\theta_h}{C_h+\Delta} - \theta_l)$, there is full information revelation in a one-period model.*

According to Lemma 3, if α is sufficiently small, only a separating equilibrium exists in a one-period model. If the principal in the good market does not reveal information and offers incentive pay of $\frac{C_h}{\theta_h}$ to the agent, the principal in the bad market will mimic him by offering the same contract. This is because the agent will ask for incentive pay of $\frac{C_l}{\theta_l}$ if the principal in the bad market reveals the bad market condition. Pooling of principals across two market conditions leads to high incentive pay if there are many principals in the bad market. Although the principal in the good market needs to incur a salary of Δ to separate from the principal in the bad market, the cost of information revelation is smaller than the incentive cost due to pooling.

Under the two-period framework, this result no longer holds. Although early adaptation features full information revelation in the first period, the contract reveals the market condition only following good performance in the second period. The intuition is as follows. A firm that operates in a good market in the first period commits to no information revelation (or a high incentive cost) in the second period following bad performance to reduce the information revelation cost in the first period, as such commitment is more costly for the principal in a bad market to offer. Early information revelation therefore limits a firm's ability to reveal information in the future.

Specifically, two forces work in the opposite directions. I consider an extreme scenario in which α and β approach zero for simple illustration. This is the environment in which full information revelation (SSS) is most likely to arise according to Lemma 3, but the principal chooses early adaptation over full information revelation. Not revealing information following bad performance increases the *expected* second-period incentive cost of a firm in a good market by an amount of $(1-\theta_h)(C_h - C_l)$. However, a firm that operates in a good market in the first period is less likely than a firm in a bad market to achieve bad performance. As a result, the reduction in the cost of first-period information revelation, which is $(1-\theta_l)(C_h - C_l)$, outweighs the increase in incentive cost by an amount of $(\theta_h - \theta_l)(C_h - C_l)$.

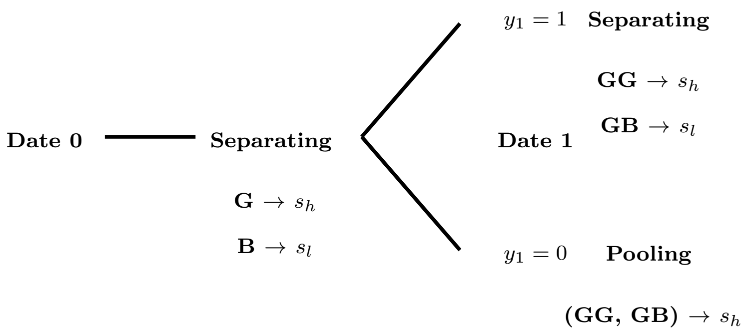


Figure 4: Adaptation Path under Early Adaptation

Figure 5 presents the adaptation path under early adaptation. The agent implements the strategy that adapts to the market condition in the first period. The information environment improves following good performance but worsens following bad performance. If first-period output is high, she then, in the second period, implements the strategy that adapts to the new market condition. While firms in GG continue implementing the high-cost strategy following good performance, firms in GB have to act more conservatively, for instance, cutting costs, orienting businesses to local markets, becoming technologically moderate, etc.

However, there is inertial adoption of the high-cost strategy following bad performance. Early adaptation limits a firm's ability to adapt in the future. If the first-period output is low, the agent then implements the high-cost strategy because the compensation plan chosen following bad performance is not indicative of the market condition. That is, agents in both GB and GG adopt the high-cost strategy due to pooling. The firm in GB therefore accumulates inertia in adopting the high-cost strategy following bad performance. As argued above, the principal in a good market in the first period commits to a contract that promises high-powered incentive pay in the second period following bad performance to reduce the cost of information revelation in the first period. Because the agent will not abandon high-powered incentive pay once she learns either the good or bad market condition, the principal has no incentive to further reveal the market condition.

4.2 Late Adaptation

To help illustrate the intuition for late adaptation, I first present an auxiliary result in Lemma 4. According to Lemma 4, if α is sufficiently large, only a pooling equilibrium exists in a one-period model, as the information revelation cost is high.

Lemma 4 *If $\alpha \geq 1/(\theta_h - \theta_l)(\frac{C_h\theta_h}{C_h+\Delta} - \theta_l)$, there is no information revelation in a one-period model.*

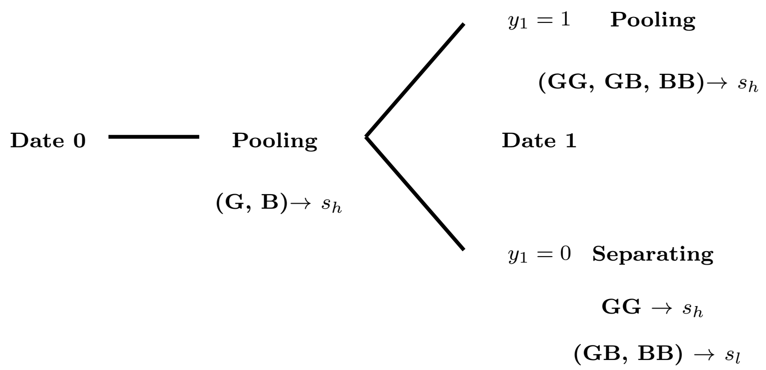


Figure 5: Adaptation Path under Late Adaptation

Under the two-period framework, this result no longer holds. Although late adaptation features no information revelation in the first period, the chosen compensation plan reveals the market condition following bad performance in the second period. The intuition is as follows. Although the agent is not able to precisely learn m_1 from the contract, she attempts to infer it from her past performance, as she implements the high-cost strategy in the first period.

$$Pr(m_1 = G|y_1 = 0) = \frac{\alpha(1-\theta_h)}{\alpha(1-\theta_h) + (1-\alpha)(1-\theta_l)} < \alpha \tag{4.1}$$

As shown in Equation 4.1, bad past performance reinforces the agent's negative belief concerning the market condition. However, such inference works to the disadvantage of a firm operating in a good market in the first period, as it is more likely for such a firm to remain in a good market than it is for a firm in a bad market to find itself in a good market. Because not revealing the market condition through contracts would cause a substantial increase in the cost of incentivizing the agent to implement s_h, the principal, if she continues to operate in the good market, chooses to reveal the market condition following bad performance.

$$Pr(m_1 = G|y_1 = 1) = \frac{\alpha\theta_h}{\alpha\theta_h + (1-\alpha)\theta_l} > \alpha \tag{4.2}$$

On the contrary, good past performance reinforces the agent's positive belief concerning the market condition, as shown in Equation 4.2. Such inference works to the advantage of a firm operating in a good market. Because it is more likely for a firm in a good market to remain in a good market than it is for a firm operating in a bad market to find itself in a good market, not revealing the market condition through contracts would not cause an increase in the cost of incentivizing the agent to implement s_h that is greater than the cost of information revelation.

Figure 5 presents the adaptation path under late adaptation. The agent implements the high-cost strategy in the first period. In contrast with early adaptation, the information environment improves following bad performance but worsens following good performance. Following good performance, firms operating in a bad market induce their agents to take s_h at a lower cost in the second period by pooling with firms in a good market. They take advantage of the agent's incorrect inference. This result suggests that inertial adoption of the high-cost strategy is more likely to arise when firms are performing well. The principal in a bad market has no incentive to reveal a bad market condition, as it demotivates the agent.

However, if first-period output is low, the agent then, in the second period, implements the strategy that adapts to the new market condition. While firms in GG continue implementing the high-cost strategy following good performance, firms in GB and BB have to act more conservatively, for instance, cutting costs, orienting businesses to local markets, becoming technologically moderate, etc. This is also reminiscent of the Schumpeterian view that economic downturns play a positive role in promoting long-run productivity growth.

4.3 Remarks

Which information revelation structure is more likely to arise in equilibrium? If the ex-ante probability of a good market condition is high (α is high) and the probability of market deterioration is low (or β is high), then late adaptation occurs. If both are low, early adaptation occurs. In particular, as β increases, early adaptation no longer survives as the equilibrium. Intuitively, a volatile market condition (or β is low) makes a firm more willing to inform employees of market changes and to motivate adaptation, as it increases the incentive cost of pooling relative to the cost of information revelation. In contrast, a stable market outlook (or β is high) makes a principal with good news less willing to build an informative incentive system early on, as she can pool with the other type at a lower cost later. My model therefore predicts a cohort effect of the initial market prospects on shaping a firm's adaptation path.

The intuition is as follows. Early adaptation arises if a firm is founded when a good market condition is perceived to be highly transitory (or β is low). Because the agent believes that there is a very high probability of being in a bad market in the second period and thus of achieving a failure, commitment to pooling at the high-cost strategy after an initial failure means a high-powered incentive plan that is costly for both principals with either good or bad news. If a good market condition is instead perceived to be highly persistent (or β is high), the principal with good news can now pool with the principal with bad news at lower incentive pay. This is because the agent believes that there is a very low probability of being in a bad market in the second period and thus of achieving a failure. Therefore, instead of offering a high-powered incentive plan, the principal with good news offers reduced incentive pay, which is no longer costly for the principal with bad news to offer.

However, if the initial market condition is very likely to be good (high α) and the market change is highly transitory (low β), early adaptation can still survive as an equilibrium but only if the cost of implementing the high-cost strategy is very high relative to that of implementing the low-cost strategy. Intuitively, if α is high, only a high C_h relative to C_l can deter the firm in a bad market from pooling at the high-cost strategy. If C_h is not sufficiently high, the cost of information revelation in the first period outweighs the reduction in incentive cost, which gives rise to late adaptation. If one interprets the difference between C_h and C_l as how drastic the strategic change is, this model also implies that early adaptation is more likely to occur if the new market condition requires a drastic strategic change.

To summarize, equilibrium contracts do not fully reveal market conditions. Full information revelation is so costly that a firm sometimes chooses not to inform the agent of market changes. As a result, the equilibrium path of adaptation, either in the form of early or late adaptation, entails adaptation failure. Depending on underlying economic prospects, inertial implementation of an old strategy, a consequence of insufficient information revelation, may occur following either success or failure.

5 Equilibrium Compensation Structure

The previous section investigates the equilibrium structure of information revelation. In this section, I characterize the equilibrium contract that supports each adaptation path.

5.1 Benchmark – A Fully Revealing Contract

Before I present equilibrium contracts, I first characterize a fully revealing contract in this benchmark case. Such a contract reveals the principal's private information at any time t and following any performance level. To be more precise, I impose truth-telling constraints onto the principal's maximization programme in both periods and following any performance levels.

Below is the maximization programme for a principal who privately knows a good market condition at date 0 and is forced to reveal information throughout.

$$\max_{\mathcal{M}} \quad \theta_h\{\beta(\theta_h(2-w_{11}^{GG})+(1-\theta_h)(1-w_{10}^{GG}))+(1-\beta)(\theta_l(2-w_{11}^{GB})+(1-\theta_l)(1-w_{10}^{GB}))\}$$
$$+(1-\theta_h)\{\beta(\theta_h(1-w_{01}^{GG})+(1-\theta_h)(0-w_{00}^{GG}))+(1-\beta)(\theta_l(1-w_{01}^{GB})+(1-\theta_l)(0-w_{00}^{GB}))\}$$

The maximization programme is subject to several constraints. To save space, I do not list the limited liability constraints.[18]

$$s.t. \quad \theta_h w_{11}^{GG}+(1-\theta_h)w_{10}^{GG}-C_h \geq w_{10}^{GG} \tag{5.1}$$
$$\theta_h w_{01}^{GG}+(1-\theta_h)w_{00}^{GG}-C_h \geq w_{00}^{GG} \tag{5.2}$$
$$\theta_l w_{11}^{GB}+(1-\theta_l)w_{10}^{GB}-C_l \geq w_{10}^{GB} \tag{5.3}$$
$$\theta_l w_{01}^{GB}+(1-\theta_l)w_{00}^{GB}-C_l \geq w_{00}^{GB} \tag{5.4}$$
$$\theta_h\{\beta(\theta_h(w_{11}^{GG}-w_{10}^{GG})+w_{10}^{GG}-C_h)+(1-\beta)(\theta_l(w_{11}^{GB}-w_{10}^{GB})+w_{10}^{GB}-C_l)\} \tag{5.5}$$
$$+(1-\theta_h)\{\beta(\theta_h(w_{01}^{GG}-w_{00}^{GG})+w_{00}^{GG}-C_h)+(1-\beta)(\theta_l(w_{01}^{GB}-w_{00}^{GB})+w_{00}^{GB}-C_l)\}$$
$$-C_h \geq \beta(\theta_h(w_{01}^{GG}-w_{00}^{GG})+w_{00}^{GG}-C_h)+(1-\beta)(\theta_l(w_{01}^{GB}-w_{00}^{GB})+w_{00}^{GB}-C_l)$$
$$\theta_l(2-w_{11}^{GB})+(1-\theta_l)(1-w_{10}^{GB}) \geq \theta_l(2-w_{11}^{GG})+(1-\theta_l)(1-w_{10}^{GG}) \tag{5.6}$$
$$\theta_l(1-w_{01}^{GB})+(1-\theta_l)(0-w_{00}^{GB}) \geq \theta_l(1-w_{01}^{GG})+(1-\theta_l)(0-w_{00}^{GG}) \tag{5.7}$$
$$\theta_l^2(2-w_{11}^{BB})+\theta_l(1-\theta_l)(1-w_{10}^{BB})+\theta_l(1-\theta_l)(1-w_{01}^{BB})+(1-\theta_l)^2(0-w_{00}^{BB}) \tag{5.8}$$
$$\geq \theta_l^2(2-w_{11}^{GB})+\theta_l(1-\theta_l)(1-w_{10}^{GB})+\theta_l(1-\theta_l)(1-w_{01}^{GB})+(1-\theta_l)^2(0-w_{00}^{GB})$$

Lemma 2 is very useful in simplifying the agent's project choice constraints of implementing s_l or s_h. If the agent learns of a good market condition, then she implements the high-cost strategy. If the agent learns of a bad market condition, she implements the low-cost strategy. I can thus focus only on the incentive constraints that ensure that the agent works. Constraints 5.1-5.5 are the agent's incentive constraints, under which the agent adopts the adaptive strategy rather than shirking. If output is low, the principal cannot distinguish whether the agent works or not. One could also easily verify that, due to limited liability, the agent's binding incentive constraints imply non-binding participation constraints under the assumption of zero reservation utility.[19] Constraints 5.6-5.8 are the principal's truth-telling constraints.

Proposition 3 ***A fully revealing contract.***
If $m_1 = G$, the principal commits to such a contract that restricts her to only two

[18] All contingent payments must be greater than or equal to zero.

[19] In fact, under information symmetry, because the probability of success is assumed to be zero if the agent shirks and w_{00}^{GG} is equal to zero, the incentive constraint of 5.2 is also the agent's participation constraint following good performance in a non-deteriorating market.

compensation plans to choose from in the second period:

1. *Compensation plan one:* $w_{00}^{GB} = \Delta$, $w_{01}^{GB} = \frac{C_l}{\theta_l} + \Delta$, $w_{10}^{GB} = \frac{C_h}{\theta_h} + \Delta$, *and* $w_{11}^{GB} = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_h} + \Delta$;
2. *Compensation plan two:* $w_{00}^{GG} = 2\Delta$, $w_{01}^{GG} = \frac{C_l}{\theta_l} + 2\Delta$, $w_{10}^{GG} = \frac{C_h}{\theta_h} + 2\Delta$, *and* $w_{11}^{GG} = 2\frac{C_h}{\theta_h} + 2\Delta$.

If $m_1 = B$, *the principal offers the same contract as described in Proposition 1.*

Proposition 3 indicates three interesting features of a fully separating contract. First, as in the first benchmark case, when the agent is informed of market changes, the firm achieves full adaptation. A fully revealing contract thus incentivizes the agent to implement the strategy that adapts to the current market condition.

Second, neither of the two compensation plans contains long-term incentive pay that depends on both y_1 and y_2. The two plans can be decomposed into a fixed component (salary) and short-term performance-based pay (bonus). The equilibrium contracts, as shown in the next section, do not possess this feature because the equilibrium structure of information revelation is path-dependent. Note that the signalling cost of information revelation in this paper is in the form of money transfers from the principal to the agent rather than a pure waste of money burning as in Akerlof (1970).

Corollary 2 *The contract in Proposition 3 can be further characterized by a downward rigid contract. The principal commits to paying:*

- *in* $t = 1$, $w_1^G = \frac{C_h}{\theta_h} + \frac{1}{2}\Delta$ *if* $y_1 = 1$ *and* $w_0^G = \frac{1}{2}\Delta$ *if* $y_1 = 0$;
- *in* $t = 2$, *either* $w_1^B = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_h} + \frac{1}{2}\Delta$ *if* $y_2 = 1$ *and* $w_0^B = \frac{1}{2}\Delta$ *if* $y_2 = 0$, *or* $w_1^G = \frac{C_h}{\theta_h} + \frac{3}{2}\Delta$ *if* $y_2 = 1$ *and* $w_0^G = \frac{3}{2}\Delta$ *if* $y_2 = 0$.

Third, although the two compensation plans can be replicated with short-term contracts, the contract offered in the first period cannot. Essentially, the principal commits to a long-term contract that promises non-decreasing compensation in the second period. Corollary 2 illustrates this feature. w_0^G is the salary, and $w_1^G - w_0^G$ is the bonus for good performance in the first period. Salary and bonus in the second period can be decomposed in the same way. Figure 6 presents a graphical illustration of the downward rigid contract in Corollary 2. Red colour indicates a constantly good market and blue a deteriorating market. Dashed lines represent salary and solid lines represent a bonus following good performance. The principal raises either the salary in the second period if the market remains good or the bonus if the market condition deteriorates.

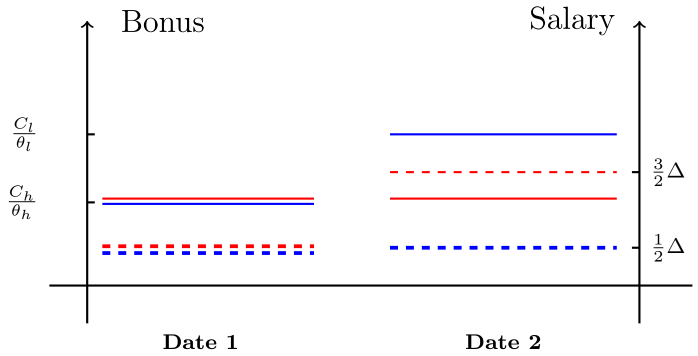


Figure 6: A fully separating contract

Note: Dashed line – salary; Solid line–bonus; Red: GG; Blue: GB.

5.2 Contract under Early Adaptation

Proposition 4 describes the contract under early adaptation. The principal in a deteriorating market, following bad performance, offers the same compensation plan as the principal in a stable market, and thus there is no information revelation following bad performance. Define that $\bar{\theta} = \alpha\theta_h + (1-\alpha)\theta_l$.

Proposition 4 ***Compensation structure under early adaptation.***

- *If $m_1 = G$, the principal commits to the following contract that restricts her to choosing from only two compensation plans in the second period:*

  1. *Compensation plan one:* $w_{00}^{GB} = (1-\theta_l)(C_l - C_h\theta_l/\bar{\theta}) + \Delta + \beta\theta_l\Delta$, $w_{10}^{GB} = \frac{C_h}{\theta_h} - \beta\Delta + w_{00}^{GB}$, $w_{01}^{GB} = w_{00}^{GB} + C_h/\bar{\theta}$, *and* $w_{11}^{GB} = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_h} - \beta\Delta + w_{00}^{GB}$*;*
  2. *Compensation plan two:* $w_{00}^{GG} = (1-\theta_l)(C_l - C_h\theta_l/\bar{\theta}) + \Delta + \beta\theta_l\Delta$, $w_{10}^{GG} = \frac{C_h}{\theta_h} + (1-\beta)\Delta + w_{00}^{GG}$, $w_{01}^{GG} = w_{00}^{GG} + C_h/\bar{\theta}$, *and* $w_{11}^{GG} = 2\frac{C_h}{\theta_h} + (1-\beta)\Delta + w_{00}^{GG}$.

- *If $m_1 = B$, the principal offers the same contract as described in Proposition 1.*

The principal offers the first compensation plan if the market condition deteriorates and the second if it does not. The compensation structure in Proposition 4 exhibits two interesting features. First, performance-based pay is path-dependent and cannot be replicated by two short-term performance payments. More explicitly, following bad interim performance, the agent receives an extra amount of $w_{01}^{GB} - w_{00}^{GB} = w_{01}^{GG} - w_{00}^{GG} = C_h/\bar{\theta}$ if she achieves high output. However, following good performance, the agent either receives an extra amount of $w_{11}^{GB} - w_{10}^{GB} = \frac{C_l}{\theta_l}$ or $w_{11}^{GG} - w_{10}^{GG} = \frac{C_h}{\theta_h}$ if she achieves high output. Rewards for high output in the second period following good and bad performance are not the same.

The use of long-term equity under information asymmetry is a direct implication of path-dependent information revelation. As argued in the previous section, revealing the market condition only following good performance reduces the principal's signalling cost in the first period to an extent that outweighs the increase in the incentive cost following bad performance. Therefore, the reward for high performance in the second period is higher following bad performance than following good performance ($C_h/\bar{\theta} = w_{01}^{GG} - w_{00}^{GG} > w_{11}^{GG} - w_{10}^{GG} = \frac{C_h}{\theta_h}$).

In contrast, the two benchmark cases show that rewards for high output in the second period following good and bad performance are the same. In the first benchmark case, the principal in a good market does not need to inform, as both the principal and the agent are symmetrically informed of the market condition. In the second benchmark of information asymmetry, truth-telling constraints are imposed to ensure full information revelation and full adaptation. Neither of the two cases involves path-dependent information revelation nor requires the use of long-term incentive pay.

The second interesting feature of this contract is that, as shown in Corollary 3, it can also be implemented by a downward rigid contract.

Corollary 3 *Contract in Proposition 4 can be implemented by a downward rigid structure:*

- *In $t = 1$, $w_0^G = w_{00}^{GB}/2 - \beta\Delta/2$ if $y_1 = 0$, $w_1^G = \frac{C_h}{\theta_h} + w_0^G$ if $y_1 = 1$.*
- *In $t = 2$, following $y_1 = 0$, $w_0^G = w_0^B = w_{00}^{GB}/2 + \beta\Delta/2$ if $y_2 = 0$, $w_1^G = w_1^B = C_h/\bar{\theta} + w_0^G$ if $y_2 = 1$.*
- *In $t = 2$, following $y_1 = 1$, the principal commits to paying either*
  - *$w_0^G = \Delta + w_{00}^{GB}/2 - \beta\Delta/2$ if $y_2 = 0$, $w_1^G = \frac{C_h}{\theta_h} + w_0^G$ if $y_2 = 1$; Or*
  - *$w_0^B = w_{00}^{GB}/2 - \beta\Delta/2$ if $y_2 = 0$, $w_1^B = \frac{C_l}{\theta_l} + w_0^B$ if $y_2 = 1$.*

Corollary 3 suggests that contract under early adaptation is non-decreasing in both salary and performance-based pay. Figure 7 offers a graphical illustration of the compensation structure. The principal offers a salary of $w_{00}^{GB}/2 - \beta\Delta/2$ in the first period. If the market remains good, the principal offers $w_{00}^{GB}/2 + \beta\Delta/2$ following bad performance and $\Delta + w_{00}^{GB}/2 - \beta\Delta/2$ following good performance. If the market worsens, the principal also offers $w_{00}^{GB}/2 + \beta\Delta/2$ following bad performance due to no information revelation and $w_{00}^{GB}/2 - \beta\Delta/2$ following good performance. The increase in salary is most pronounced following good performance and in a stable market, a situation in which the principal continues to reveal information.

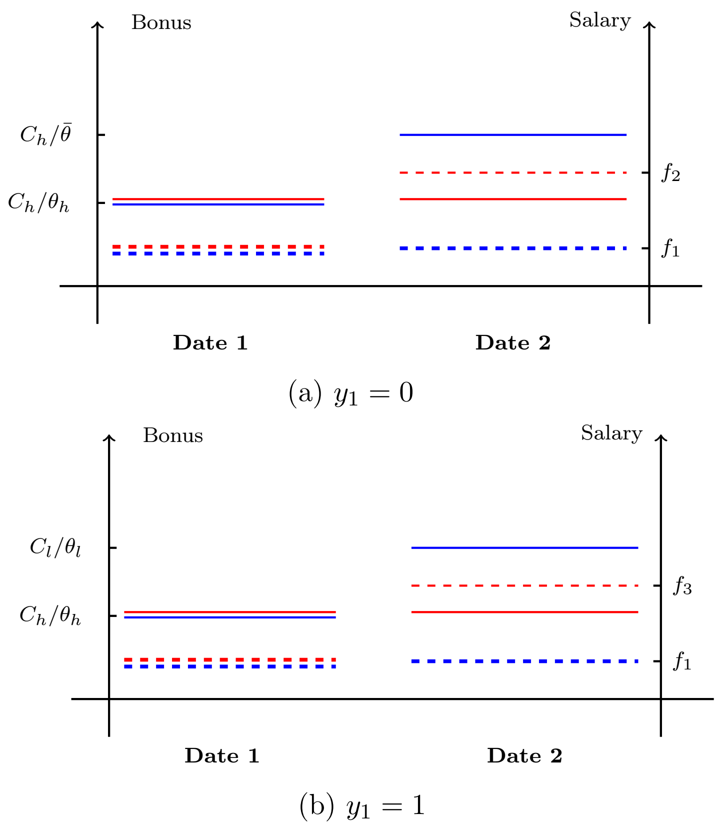


Figure 7: Contract under Early Adaptation

Note: Dashed line – salary; Solid line–bonus; Red: GG; Blue: GB.
$\Delta = C_l - \frac{\theta_l}{\theta_h} C_h$, $f_1 = w_{00}^{GB}/2 - \beta\Delta/2$, $f_2 = w_{00}^{GB}/2 + \beta\Delta/2$, and $f_3 = w_{00}^{GB}/2 - \beta\Delta/2 + \Delta$.

Performance-based pay increases from $\frac{C_h}{\theta_h}$ in the first period to $C_h/\bar{\theta}$ in the second period following bad performance. This is to motivate the agent to implement the high-cost strategy under pooling. It remains the same in the second period following good performance in a non-deteriorating market but increases to $\frac{C_l}{\theta_l}$ in a deteriorating market, as information revelation following good performance allows the principal to offer incentive pay to induce the adaptive strategy to be implemented.

5.3 Contract under Late Adaptation

Proposition 5 describes the contract under late adaptation. The principal in a good market does not reveal the market condition in the first period or in the second period following good performance. First define $\alpha' = Pr(\theta_h|y_1 = 1) = \alpha\theta_h/(\alpha\theta_h + (1-\alpha)\theta_l)$ and $\bar{\theta}_\alpha = \alpha' q\theta_h + (1-\alpha' q)\theta_l$.

Proposition 5 ***Compensation structure under late adaptation.***

- *If $m_1 = G$, the principal commits to the following contract that restricts her to choosing from only two compensation plans in the second period:*
    1. *Compensation plan one:* $w_{00}^{GB} = 0$, $w_{01}^{GB} = \frac{C_l}{\theta_l}$, $w_{10}^{GB} = \frac{1}{\theta_\alpha}\{(\theta_\alpha + 1)C_h + \alpha\beta\theta_h\Delta - ((\alpha\theta_h\bar{\theta} + (1-\alpha)\theta_l^2))C_h/\bar{\theta}_\alpha\}$, $w_{11}^{GB} = C_h/\bar{\theta}_\alpha + w_{10}^{GB}$;
    2. *Compensation plan two:* $w_{00}^{GG} = \Delta$, $w_{01}^{GG} = \frac{C_h}{\theta_h} + \Delta$, $w_{10}^{GG} = \frac{1}{\theta_\alpha}\{(\theta_\alpha + 1)C_h + \alpha\beta\theta_h\Delta - ((\alpha\theta_h\bar{\theta} + (1-\alpha)\theta_l^2))C_h/\bar{\theta}_\alpha\}$, $w_{11}^{GG} = C_h/\bar{\theta}_\alpha + w_{10}^{GG}$.
- *If $m_1 = B$, the principal offers the same contract.*

The principal in a good market in the first period offers the first compensation plan if the market condition deteriorates and the second if it does not. The principal in a bad market in the first period offers the same contract but chooses the first compensation plan. The compensation structure in Proposition 4 also exhibits two interesting features.

First, as in early adaptation, performance-based pay is also path-dependent and cannot be replicated by two short-term performance payments. More explicitly, following good performance, the agent receives an extra amount of $w_{11}^{GG} - w_{10}^{GG} = w_{11}^{GB} - w_{10}^{GB} = C_h/\bar{\theta}_\alpha$ if she achieves high output in the second period. However, following bad performance, the agent either receives an extra amount of $w_{01}^{GB} - w_{00}^{GB} = \frac{C_l}{\theta_l}$ or $w_{01}^{GG} - w_{00}^{GG} = \frac{C_h}{\theta_h}$ if she achieves high output in the second period. The reward for high output in the second period following good or bad performance is not the same.

The intuition for this feature is similar to early adaptation. The use of long-term equity under information asymmetry is also a direct implication of path-dependent information revelation. As argued in the previous section, revealing the market condition following bad performance saves the principal's incentive cost of pooling in the first period. Therefore, the reward for high performance in the second period is higher following good performance than following bad performance($C_h/\bar{\theta}_\alpha = w_{11}^{GG} - w_{10}^{GG} > w_{01}^{GG} - w_{00}^{GG} = \frac{C_h}{\theta_h}$).

In contrast to early adaptation, Corollary 4 shows that a downward rigid contract cannot always be implemented in late adaptation. Figure 8 offers an graphical illustration of the compensation structure.

Corollary 4 *Contract in Proposition 5 can be implemented in the following form:*

- *In $t = 1$, $w_1^G = w_1^B = w_{10}^{GB}$ if $y_1 = 1$, $w_1^G = w_1^B = 0$ if $y_1 = 0$.*
- *In $t = 2$, following $y_1 = 1$, $w_1^G = w_1^B = C_h/\bar{\theta}_\alpha$ if $y_2 = 1$, $w_0^G = w_0^B = 0$ if $y_2 = 0$.*
- *In $t = 2$, following $y_1 = 0$, the principal commits to paying either*
  - *$w_0^G = \Delta$ if $y_2 = 0$, $w_1^G = \frac{C_h}{\theta_h} + w_0^G$ if $y_2 = 1$; Or*
  - *$w_0^B = 0$ if $y_2 = 0$, $w_1^B = \frac{C_l}{\theta_l}$ if $y_2 = 1$.*

If α and β are sufficiently large, under which late adaptation is mostly likely to be the equilibrium, performance pay in the first period is greater than that in the second period $w_{10}^{GG} > w_{01}^{GG} - w_{00}^{GG}$ and $w_{10}^{GG} > w_{11}^{GG} - w_{10}^{GG}$.[20]

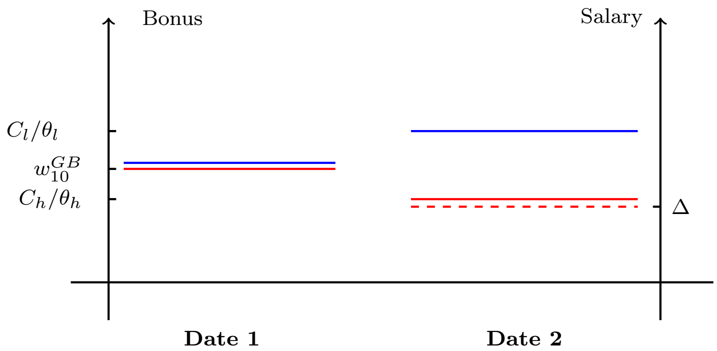
The contract in late adaptation does not reveal the market condition in the first period and only does so in the second period following good performance. Intuitively, to motivate the agent to implement the high-cost strategy under pooling in the first period, the principal in a good market has to provide high performance-based pay, which explains $w_{10}^{GG} > w_{01}^{GG} - w_{00}^{GG}$. Because the agent could receive a salary even following bad performance due to information revelation, the principal has to pay an even higher incentive reward to induce first-period effort, which explains $w_{10}^{GG} > w_{11}^{GG} - w_{10}^{GG}$.

6 Discussion

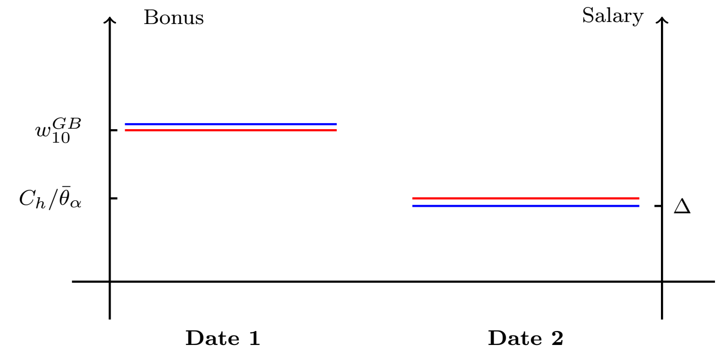
6.1 Renegotiation

In this section, I consider renegotiation of contracts at the beginning of the second period. The renegotiation is led by the informed principal and corresponds to renegotiation in common value situations, as the principal's private information enters into

[20]Equivalently, $w_{10}^{GB} > w_{11}^{GB} - w_{10}^{GB}$.



(a) $y_1 = 0$



(b) $y_1 = 1$

Figure 8: Contract under Late Adaptation

Note: Dashed line – salary; Solid line–bonus; Red: GG; Blue: GB.
$\Delta = C_l - \frac{\theta_l}{\theta_h} C_h$.

the agent's utility function. I first introduce two concepts regarding renegotiation-proofness as in Maskin and Tirole (1992).

Definition Weak form renegotiation-proofness. A contract is **weakly** renegotiation-proof, if there exist no other incentive-compatible contracts that (i) make at least one type of principals better off, and (ii) yield the agent at least as much utility.

Definition Strong form renegotiation-proofness. A contract is **strongly** renegotiation-proof if and only if it is renegotiation-proof when the uninformed party leads the renegotiation.

In other words, a contract that is weakly renegotiation-proof provides the principal in either type of markets the highest possible profit given that it is incentive compatible. A contract that is strongly renegotiation-proof also provides the agent in either type of markets the highest possible utility. If the principal who initiates renegotiation is at the indifference point of having no incentive to renegotiate, then the contract is weakly renegotiation-proof regardless of the agent's potential utility improvement.

The principal that operates in the bad market prevents the principal that operates in the good market from truthfully revealing good information at zero cost. This intuition is robust to any concepts of renegotiation-proofness. However, since the long-term contracts may involve commitment made by the principal in the good market, the principal may have the incentive to renegotiate in the second period. Proposition 6 discusses the renegotiation-proofness of contracts that induce early and late adaptation.

Proposition 6

- *The optimal contract that induces late adaptation is both weakly and strongly renegotiation-proof;*
- *The optimal contract that induces early adaptation is weekly renegotiation-proof, it is also strongly renegotiation-proof if* $\frac{\theta_h-\theta_l}{\theta_l}(1-\frac{C_h}{\theta_l})-\frac{C_h-C_l}{\theta_l}>0$.

The reason why the contract that induces early adaptation is weakly but not strongly renegotiation-proof is that the principal in the good market cannot improve his profit by revealing good information to the agent. The agent will not give up the highly-powered incentive package. The principal is exactly at the point of having no incentive to renegotiate. If the agent leads the renegotiation, then he can commit to a set of new contracts that saves the cost effort in bad condition by inducing the principals in different markets select different contracts. If $\frac{\theta_h-\theta_l}{\theta_l}(1-\frac{C_h}{\theta_l})-\frac{C_h-C_l}{\theta_l}<0$, the agent could not find a set of contracts that satifies both: 1) the principal in a deteriorating market will choose one contract; 2) the principal in a constantly good market will not choose the contract chosen by the prinicpal in a deteriorating market.

6.2 Government Intervention

Previous sections show that information asymmetry leads to a failure of information revelation and insufficient adaptation. This section discusses two policies: a government could apply a subsidy rate or a tax rate to facilitate or direct adaptation. A firm is entitled to a subsidy if it achieves good performance and is charged a tax only if it achieves bad performance. Both the subsidy and tax are rate-based and are applied to a firm's after-compensation earnings.

The *selective* policies create differential effects on a firm in a good market and a firm in a bad market. Specifically, the effective subsidy awarded to the firm in a good market is higher than that awarded to the firm in a bad market, and the effective tax levied on the firm in a bad market is higher than that in a good market. The policies alleviate the truth-telling constraints in the sense that they make it more costly for the firm in a bad market to mimic. The signalling cost is reduced for the firm in a good market, and it is thus more willing to choose the socially efficient contract. In a one-period model, the signalling cost is reduced from Δ to $\Delta/(\frac{1-t\theta_l}{1-t})$ if a subsidy rate t is applied and to $\Delta/(1+t(1-\theta_l))$ if a tax rate t is applied.

In the following analysis, I consider the tax policy for a detailed illustration under early adaptation. Lemma 5 and Lemma 6 present the compensation structure under the second benchmark case, in which truth-telling constraints are imposed, and the compensation structure in early adaptation.

Lemma 5 ***A fully revealing contract under bad-performance tax***
If $m_1 = G$, the principal commits to such a contract that restricts her to only two compensation plans to choose from in the second period:

1. *Compensation plan one: $w_{00}^{GB} = \Delta/(1+t(1-\theta_l)^2)$, $w_{01}^{GB} = \frac{C_l}{\theta_l} + w_{00}^{GB}$, $w_{10}^{GB} = \frac{C_h}{\theta_h} + w_{00}^{GB}$, and $w_{11}^{GB} = \frac{C_l}{\theta_l} + w_{10}^{GB}$;*
2. *Compensation plan two: $w_{00}^{GG} = \Delta/(1+t(1-\theta_l)^2) + \Delta/(1+t(1-\theta_l))$, $w_{01}^{GG} = \frac{C_l}{\theta_l} + w_{00}^{GG}$, $w_{10}^{GG} = \frac{C_h}{\theta_h} + w_{00}^{GG}$, and $w_{11}^{GG} = \frac{C_h}{\theta_h} + w_{10}^{GG}$.*

If $m_1 = B$, the principal offers the same contract as described in Proposition 1.

Lemma 5 shows that both w_{00}^{GB} and w_{00}^{GG} are reduced compared to the level under zero tax. The bad-performance tax alleviates the signalling problem imposed by information asymmetry.

Lemma 6 ***Early adaptation contract under bad-performance tax***

- *If $m_1 = G$, the principal commits to the following contract that restricts her to choosing from only two compensation plans in the second period:*

  1. *Compensation plan one:* $w_{00}^{GB} = ((1-\theta_l)(C_l - C_h\theta_l/\bar{\theta}) + \Delta + \beta\theta_l\Delta)/(1 + t(1-\theta_l)^2)$, $w_{10}^{GB} = \frac{C_h}{\theta_h} - \beta\Delta/(1+t(1-\theta_l)^2) + w_{00}^{GB}$, $w_{01}^{GB} = C_h/\bar{\theta} + w_{00}^{GB}$, *and* $w_{11}^{GB} = \frac{C_l}{\theta_l} + w_{10}^{GB}$*;*
  2. *Compensation plan two:* $w_{00}^{GG} = ((1-\theta_l)(C_l - C_h\theta_l/\bar{\theta}) + \Delta + \beta\theta_l\Delta)/(1 + t(1-\theta_l)^2)$, $w_{10}^{GG} = \frac{C_h}{\theta_h} + \Delta/(1+t(1-\theta_l)) - \beta\Delta/(1+t(1-\theta_l)^2) + w_{00}^{GG}$, $w_{01}^{GG} = C_h/\bar{\theta} + w_{00}^{GG}$, *and* $w_{11}^{GG} = \frac{C_h}{\theta_h} + w_{10}^{GG}$.

- *If $m_1 = B$, the principal offers the same contract as described in Proposition 1.*

Lemma 6 shows that both w_{00}^{GB} and $w_{10}^{GG} - w_{10}^{GB}$ are reduced compared to the level under zero tax. The bad-performance tax alleviates the signalling problem imposed by information asymmetry.

Proposition 7 *If $t \geq \frac{\theta_h - \theta_l}{(1-\theta_h)(1-\theta_l^2)}$, the principal in a good market chooses the fully revealing contract over the early adaptation contract.*

The discussion in this section therefore highlights the form and extent of public policies that the government should implement, especially in industries experiencing upgrading and restructuring. Proposition 7 shows that there exists a minimum level of tax rate above which the informed principal in a good market adopts the fully revealing contract. The intuition is that the tax rate should be sufficiently high to make mimicking costly for a principal in a bad market. In addition, for government policies to facilitate adaptation, they need to be selective. One could easily show that taxing or subsidizing firms following both good and bad performance does not reduce the signalling cost, which sheds light on the policy debate concerning the form of government assistance.

6.3 Termination of Employment

In reality, the actual contract space includes other incentive tools in addition to the compensation scheme. For instance, the principal could dismiss the agent following bad performance. In general, the termination of employment reduces the rent that the agent could extract from the principal due to the protection of limited liability. However, as the model assumes that the probability of success is zero if the agent shirks, the rent reduction effect that would be generated by the threat of termination does not exist.

One might argue that endogenizing termination gives the principal an additional signalling device. The principal in the good market could reveal her private information by committing *not* to dismiss the agent. This signalling device, however, is only useful if the principal in a bad market does not use it. Based on the argument presented in the previous paragraph, one could easily verify that firms that operate in the bad market will not use termination, which renders committing not to terminate employment useless as a signalling device.

Counterintuitively, one might also argue that the principal in the good market can reveal her private information by committing to dismiss the agent following bad performance if the firm discontinues operations or if finding a replacement is costly. This is theoretically sound, as the principal in the good market is less likely to achieve bad performance than is the principal in the bad market. Committing to dismiss the agent following bad performance, therefore, incurs less profit loss. However, the intuition hinges on the assumption that there is a loss in the continuation value.

6.4 Empirical Predictions

This paper offers a rich set of empirical predictions.

Prediction 1 (Organizational structure): *Firms that are more hierarchical are more likely to encounter insufficient and inertial adaptation and difficulty in restructuring their employees' incentive system.*

This prediction is derived by comparing the model with information asymmetry to the benchmark case in which both parties know the market conditions. It is consistent with the phenomenon of the Innovator's Dilemma, first raised by Christensen (1997). Christensen uses the term Innovator's Dilemma to describe a difficult but common situation that many firms face: They are not able to sustain their adaptation ability as they grow large. This paper posits that information friction is a plausible reason for this dilemma, as it is a natural by-product of a firm that separates strategy planning from its implementation once it grows large.

Prediction 2 (Stable market conditions): *When market conditions are stable, adaptation is inefficiently delayed.*

Prediction 3 (Volatile market conditions): *When market conditions are volatile, the optimal contract induces early adaptation.*

Predictions 2 and 3 are derived from the comparative statics with respect to the parameter β in a dynamic setting. They are largely consistent with historical observations. As noted in the introduction, the hardships of the Great Depression also spurred a period of unparalleled creativity in the U.S. auto industry, accompanied with a dramatic increase in wages. In contrast, internet companies, including numerous imitators, emerged with ferocity and frequency in a period with a superb economic outlook in the 1990s. Employees received lavish stock options in lieu of cash. The internet bubble eventually burst, leading to industry-wide consolidation. However, I am unaware of empirical work that explicitly tests this prediction.

Consistent with (but not a direct test of) the three predictions, Aghion, Bloom, Lucking, Sadun, and Reenen (2015) show that decentralized firms perform better than their centralized counterparts in terms of sales, TFP and profit growth during crisis periods.

Prediction 4 (Path-dependent adaptation): *When market conditions are stable, insufficient adaptation arises after a failure. When market conditions are volatile, insufficient adaptation arises after a success.*

Holding all else constant, the exact form of path-dependency varies with the underlying economic outlook. There are different ways to interpret this prediction. For instance, when market conditions are stable, insufficient adaptation arises if the firm is hit by a negative productivity shock. When market conditions are volatile, insufficient adaptation arises if the firm is hit by a positive productivity shock. I am unaware of empirical work that directly tests this prediction.

Prediction 5 (Compensation structure): *Firms offer high salaries in the short run and high incentive pay in the long run in early adaptation; Firms offer high incentive pay in both the short and the long run in late adaptation.*

Prediction 5 is derived from the results of the equilibrium compensation structures. The compensation structure in early adaptation leads to internal resistance: Employees refuse to relinquish the old contract. The compensation structure in late adaptation features excessive use of incentive pay due to pooling. However, I am unaware of empirical work explicitly testing this prediction.

7 Conclusions

The results suggest that a firm may fail to adapt to market changes due to information asymmetry, which is common in hierarchical organizations in which the senior management is more aware of those changes. A firm needs to structure its employees' compensation contracts to both inform and motivate them to adapt. However, it is costly to credibly inform employees of those changes and convince them of the efficacy of new strategies. A failure to overturn their belief concerning changing market conditions may lead to insufficient adaptation.

Moreover, adaptation is path-dependent and inertial; depending on the distribution of market conditions, bad performance can either foster or suppress future adaptation. In fact, more volatile market conditions induce earlier adaptation and greater information revelation. Finally, the contract that induces full adaptation is non-decreasing in both salary and performance-based pay, which, however, is too costly to offer in equilibrium. Equilibrium contracts impose a legacy problem that restrains the reconfiguration of the incentive system in place and hinders adaptation.

There are many directions in which the model can be extended. The model emphasizes the situation in which the principal is correct in her expectation of market changes. However, one could also argue that the principal might only have superior information over the distribution of market changes, as she might not be entirely certain of how market trends will evolve in the future. Another extension to consider is the case in which the principal is more informed of the change in macro-economic conditions and the agent is more informed of the change in local market conditions. The principal therefore is not only concerned with transmitting her private information to the agent but also with soliciting private information from the agent. This is also a promising avenue for future theoretical and empirical explorations.

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A Appendix

Proof of Proposition 1.

Proof First, derive the contract offered by the principal under a bad market ($m_1 = B$).

$$\max_{w\{..\}} \quad \theta_l^2(2 - w_{11}^{BB}) + \theta_l(1-\theta_l)(1 - w_{10}^{BB} + 1 - w_{01}^{BB}) + (1-\theta_l)^2(-w_{00}^{BB})$$

$$\begin{aligned} s.t. \quad & \theta_l(w_{11}^{BB} - w_{10}^{BB}) \geq C_l \\ & \theta_l(w_{01}^{BB} - w_{00}^{BB}) \geq C_l \\ & \theta_l(w_{10}^{BB} - w_{00}^{BB}) \geq C_l \\ & w_{00}^{BB}, w_{10}^{BB}, w_{01}^{BB}, w_{10}^{BB} \geq 0 \end{aligned}$$

To save space, limited liability constraints will not be listed in other proofs. The first three incentive constraints ensures that the agent chooses the low cost strategy in both periods and following both high and low output. If $w_{00}^{BB} > 0$, reducing w_{00}^{BB} by an amount of ϵ could reduce w_{11}^{BB}, w_{10}^{BB} and w_{01}^{BB} by ϵ. The principal's profit could thus increase by $\theta_l^2\epsilon + 2\theta_l(1-\theta_l)\epsilon + (1-\theta_l)^2\epsilon = \epsilon$. Therefore, $w_{00}^{BB} = 0$. Solve for the maximization program, I find that $w_{01}^{BB} = \frac{C_l}{\theta_l}$, $w_{10}^{BB} = \frac{C_l}{\theta_l}$, $w_{11}^{BB} = 2\frac{C_l}{\theta_l}$ and $w_{00}^{BB} = 0$.

Second, derive the contract under a good market condition ($m_1 = G$).

$$\max_{w\{..\}} \quad \theta_h\{q(\theta_h(2 - w_{11}^{GG}) + (1-\theta_h)(1 - w_{10}^{GG})) + (1-q)(\theta_l(2 - w_{11}^{GB}) + (1-\theta_l)(1 - w_{10}^{GB}))\}$$
$$+(1-\theta_h)\{q(\theta_h(1 - w_{01}^{GG}) + (1-\theta_h)(0 - w_{00}^{GG})) + (1-q)(\theta_l(1 - w_{01}^{GB}) + (1-\theta_l)(0 - w_{00}^{GB}))\}$$

$$\begin{aligned} s.t. \quad & \theta_h(w_{11}^{GG} - w_{10}^{GG}) \geq C_h \\ & \theta_h(w_{01}^{GG} - w_{00}^{GG}) \geq C_h \\ & \theta_l(w_{11}^{GB} - w_{10}^{GB}) \geq C_l \\ & \theta_l(w_{01}^{GB} - w_{00}^{GB}) \geq C_l \\ & \theta_h(qw_{10}^{GG} + (1-q)w_{10}^{GB}) \geq C_h \end{aligned}$$

The first four incentive constraints ensures that the agent chooses the adaptive strategy in the second period, following both high and low output, and under both good and bad market conditions. The last incentive constraint ensures that the agent choose the high-cost strategy in the first period. One can easily verify that if $m_1 = G$ and $m_2 = G$, then the principal offers a contract $w_{01}^{GG} = \frac{C_h}{\theta_h}$, $w_{10}^{GG} \geq 0$, $w_{11}^{GG} = \frac{C_h}{\theta_h} + w_{10}^{GG}$ and $w_{00}^{GG} = 0$. If $m_1 = G$ and $m_2 = B$, then the principal offers a contract $w_{01}^{GB} = \frac{C_l}{\theta_l}$, $w_{10}^{GB} \geq 0$, $w_{11}^{GB} = \frac{C_l}{\theta_l} + w_{10}^{GB}$ and $w_{00}^{GB} = 0$.

Given that $qw_{10}^{GG} + (1-q)w_{10}^{GB} = \frac{C_h}{\theta_h}$, with out loss of generality, set $w_{10}^{GB} = \frac{C_h}{\theta_h}$. Therefore, $w_{11}^{GB} = \frac{C_l}{\theta_l} + w_{10}^{GB} = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_h}$, and $w_{11}^{GG} = \frac{C_h}{\theta_h} + w_{10}^{GG} = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_h}$.

Last, I show that if agent knows the market condition, it is optimal for the firm to incentivise the agent to take the adaptive strategy. The argument is simple. If the market condition is good and the principal offers $\frac{C_l}{\theta_l}$ to incentivise the agent to implement the low-cost strategy instead of $\frac{C_h}{\theta_h}$ that encourages the implementation of the high-cost strategy, the profit would decrease by $-(\theta_h - \theta_l) + (C_h - C_l) < 0$. Please refer to the proof of Lemma 2 for a detailed discussion. Q.E.D.

Proof of Corollary 1.

Proof Readers could easily verify that the one-period short term contracts given in Corollary 1 induces the agent to implement the adaptive strategy and generate the same profit for the prinicpal. Q.E.D.

Proof of Lemma 1.

Proof One can verify that among all possible information revelation structures in Figure 3, the second-period incentive constraint for an agent in a deteriorating market is isomorphic to the one for an agent in a constantly bad market. It is

$$\theta_l(w_{01}^{XB} - w_{00}^{XB}) \geq C_l$$

One can also verify that the second-period truth-telling constraint for an agent in a deteriorating market is isomorphic to the one for an agent in a constantly bad market. It is

$$w_{00}^{XG} - w_{00}^{XB} \geq \Delta$$

If information revelation in the second period concerns only m_1, that is, if principals in GG and GB pool together and separate from principals in BB, the principal in BB will offer the same contract as GB. This is because the principal in BB in the second period has the same mimicking incentive as the principal in GB. In other words, the principal in the second period, if she reveals any private information, only reveals the second-period market condition, not the first-period market condition. Therefore separation in the second period will not occur between them. Q.E.D.

Proof of Lemma 2.

Proof If the agent knows $m_1 = G$, she will not take the market-insensitive strategy. Because $\frac{C_h}{\theta_h} < \frac{C_l}{\theta_l}$, principal will offer $\frac{C_h}{\theta_h}$ if $y = 1$. If the agent takes the market-insensitive strategy, she gets $\theta_l \frac{C_h}{\theta_h} - C_l < 0$. If she takes the market-sensitive strategy, she gets 0.

If the agent knows $m_1 = B$, she will not take the market-insensitive strategy. The principal offers $\frac{C_l}{\theta_l}$ if $y = 1$. If the agent takes the market-insensitive strategy, she gets 0. If she takes the market-sensitive strategy, she gets $\theta_l \frac{C_l}{\theta_l} - C_h < 0$.

I then take information structure SSP and verify that if the principal offers a contract which induces the agent to take the market-insensitive strategy under pooling $(w_{00}^{GX}, w_{01}^{GX})$, the principal will deviate to separate and offer a different contract which induces the agent to take the market-sensitive strategy. In other words, there exists a contract which induces pooling at the market-sensitive strategy which defeats the contract inducing pooling at the market-insensitive strategy. I do this proof for other information revelation structures in the proof of Proposition 2.

Following bad performance $y_1 = 0$, if the principal in the good market deviates and offers $w_{01}^{GG} = \frac{C_h}{\theta_h}$ and $w_{00}^{GG} = \Delta + w_{00}^{GX}$, the agent could get $\theta_h w_{01}^{GG} + (1-\theta_h) w_{00}^{GG} - C_h = \Delta + w_{00}^{GX}$, higher than $\theta_l w_{01}^{GX} + (1-\theta_l) w_{00}^{GX} - C_l = w_{00}^{GX}$ under w_{01}^{GX} and w_{00}^{GX}. The principal in the good market gets $\theta_h(1 - w_{01}^{GG}) - (1-\theta_h) w_{00}^{GG}$, which is higher than $\theta_l(1 - w_{01}^{GX}) - (1-\theta_l) w_{00}^{GX}$ under w_{01}^{GX} and w_{00}^{GX}, the difference is $(\theta_h - \theta_l)(1 - \frac{C_h}{\theta_h})$. Given that $w_{00}^{GG} = \Delta + w_{00}^{GX}$, the principal in the bad market will not want to mimic. Based on the concept of Undefeated Equilibrium, if the principal in the bad market does not follow, she will be considered as the bad type and offer a contract that fully separates himself. This contract corresponds to an annual bonus of $\frac{C_l}{\theta_l}$ and zero salary. The principal gets no less profit from this contract than $(w_{01}^{GX}, w_{00}^{GX})$ and is thus better off than $(w_{01}^{GG}, w_{00}^{GG})$. Q.E.D.

Proof of Proposition 2.

Proof Please refer to proof of Proposition 3 for the contract under SSS.
Step 1 is to characterize the contract under SSP. Define $\bar{\theta} = q\theta_h + (1-q)\theta_l$. From Lemma 2, if the agent knows the market condition, she takes the adaptive strategy which fits the external market. If the agent does not know the information, I first characterize the contract under which the agent takes the market-sensitive strategy under pooling.

$$\max_{w\{..\}} \theta_h\{q(\theta_h(2 - w_{11}^{GG}) + (1-\theta_h)(1 - w_{10}^{GG})) + (1-q)(\theta_l(2 - w_{11}^{GB}) + (1-\theta_l)(1 - w_{10}^{GB}))\}$$
$$+(1-\theta_h)\{q(\theta_h(1 - w_{01}^{GX}) + (1-\theta_h)(0 - w_{00}^{GX})) + (1-q)(\theta_l(1 - w_{01}^{GX}) + (1-\theta_l)(0 - w_{00}^{GX}))\}$$

To save space, limited liability constraints are not listed. Constraints A.9-A.4 are the agent's incentive constraints.

$$s.t.\ \theta_h(w_{11}^{GG} - w_{10}^{GG}) \geq C_h \tag{A.1}$$
$$\theta_l(w_{11}^{GB} - w_{10}^{GB}) \geq C_l \tag{A.2}$$
$$\bar{\theta}(w_{01}^{GX} - w_{00}^{GX}) \geq C_h \tag{A.3}$$
$$\theta_h\{q(\theta_h(w_{11}^{GG} - w_{10}^{GG}) + w_{10}^{GG} - C_h) + (1-q)(\theta_l(w_{11}^{GB} - w_{10}^{GB}) + w_{10}^{GB} - C_l)\} \tag{A.4}$$
$$+(1-\theta_h)\{\bar{\theta}(w_{01}^{GX} - w_{00}^{GX}) + w_{00}^{GX} - C_h)\} - C_h \geq \bar{\theta}(w_{01}^{GX} - w_{00}^{GX}) + w_{00}^{GX} - C_h$$

Constraint A.5 and A.6 are the principal's truth-telling constraints.

$$\theta_l(2 - w_{11}^{GB}) + (1-\theta_l)(1 - w_{10}^{GB}) \geq \theta_l(2 - w_{11}^{GG}) + (1-\theta_l)(1 - w_{10}^{GG}) \tag{A.5}$$
$$\theta_l^2(2 - w_{11}^{BB}) + \theta_l(1-\theta_l)(1 - w_{10}^{BB}) + \theta_l(1-\theta_l)(1 - w_{01}^{BB}) + (1-\theta_l)^2(0 - w_{00}^{BB}) \tag{A.6}$$
$$\geq \theta_l^2(2 - w_{11}^{GB}) + \theta_l(1-\theta_l)(1 - w_{10}^{GB}) + \theta_l(1-\theta_l)(1 - w_{01}^{GX}) + (1-\theta_l)^2(0 - w_{00}^{GX})$$

Rearrange the above, I get the following:

$$(A.5) \rightarrow (A.5') \qquad q\theta_h(w_{10}^{GG} - w_{00}^{GG}) + (1-q)\theta_h(w_{10}^{GB} - w_{00}^{GB}) \geq C_h$$
$$(A.6) \rightarrow (A.6') \qquad \theta_l^2 w_{11}^{GB} + \theta_l(1-\theta_l)(w_{10}^{GB} + w_{01}^{GX}) + (1-\theta_l)^2 w_{00}^{GX} \geq 2C_l$$

From above, we obtain the following compensation component expressed in w_{00}^{GX}:

$$w_{10}^{GG} \geq w_{00}^{GX} + \frac{C_h}{\theta_h} + (1-q)\Delta$$
$$w_{01}^{GX} \geq w_{00}^{GX} + \frac{C_h}{\bar{\theta}}$$
$$w_{10}^{GB} \geq w_{00}^{GX} + \frac{C_h}{\theta_h} - q\Delta$$
$$w_{11}^{GB} \geq w_{00}^{GX} + \frac{C_h}{\theta_h} + \frac{C_l}{\theta_l} - q\Delta$$
$$w_{11}^{GG} \geq w_{00}^{GX} + 2\frac{C_h}{\theta_h} + (1-q)\Delta$$

Substitute all into $A.6'$, one could solve for w_{00}^{GX}, and all other variables.

$$w_{00}^{GX} = 2C_l - \frac{\bar{\theta} + \theta_h}{\bar{\theta}\theta_h}\theta_l C_h - \theta_l(C_l - \frac{\theta_l}{\bar{\theta}}C_h) + q\theta_l\Delta$$

Step 2 is to characterize the contract under PPS. If the principal does not reveal the market condition in the first period, the agent will have to infer it from past performance following good performance. Following bad performance, the principal reveals the market condition, thus the agent does not have to infer it on her own.

$$\alpha' = Pr(\theta_1 = \theta_h | y_1 = 1) = \frac{\alpha\theta_h}{\alpha\theta_h + (1-\alpha)\theta_l} > \alpha \tag{A.7}$$

Define $\bar{\theta}_\alpha = \alpha' q\theta_h + (1-\alpha' q)\theta_l$. From Lemma 2, if the agent knows information, she takes the adaptive strategy which fits the external market. If the agent does not know the information, I first characterize the contract under which the agent takes the market-sensitive strategy under pooling.

$$\max_{w\{..\}} \ \theta_h\{q(\theta_h(2-w_{11}^{XX}) + (1-\theta_h)(1-w_{10}^{XX})) + (1-q)(\theta_l(2-w_{11}^{XX}) + (1-\theta_l)(1-w_{10}^{XX}))\}$$
$$+(1-\theta_h)\{q(\theta_h(1-w_{01}^{XG}) + (1-\theta_h)(0-w_{00}^{XG})) + (1-q)(\theta_l(1-w_{01}^{XB}) + (1-\theta_l)(0-w_{00}^{XB}))\}$$

To save space, limited liability constraints are not listed. Constraint A.8-A.11 are the agent's incentive constraints, Constraint A.12 is the agent's project choice constraint.

$$s.t. \ \bar{\theta}_\alpha(w_{11}^{XX} - w_{10}^{XX}) \geq C_h \tag{A.8}$$
$$\theta_h(w_{01}^{XG} - w_{00}^{XG}) \geq C_h \tag{A.9}$$
$$\theta_l(w_{01}^{XB} - w_{00}^{XB}) \geq C_l \tag{A.10}$$

$$\begin{aligned}
&\alpha\{\theta_h q(\theta_h(w_{11}^{XX} - w_{10}^{XX}) + w_{10}^{XX} - C_h) + (1-q)\theta_h(\theta_l(w_{11}^{XX} - w_{10}^{XX}) + w_{10}^{XX} - C_h) \\
&+(1-\theta_h)q(\theta_h(w_{01}^{XG} - w_{00}^{XG}) + w_{00}^{XG} - C_h) + (1-\theta_h)(1-q)(\theta_l(w_{01}^{XB} - w_{00}^{XB}) + w_{00}^{XB} - C_l)\} \\
&(1-\alpha)\{\theta_l(w_{11}^{XX} - w_{10}^{XX}) + w_{10}^{XX} - C_h) + (1-\theta_l)(\theta_l(w_{01}^{XB} - w_{00}^{XB}) + w_{00}^{XB} - C_l)\} - C_h \\
&\geq \alpha q(\theta_h(w_{01}^{XG} - w_{10}^{XX}) + w_{00}^{XG} - C_h) + (1-\alpha q)(\theta_l(w_{01}^{XB} - w_{00}^{XB}) + w_{00}^{XB} - C_l)
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
&\alpha\{\theta_h q(\theta_h(w_{11}^{XX} - w_{10}^{XX}) + w_{10}^{XX} - C_h) + (1-q)\theta_h(\theta_l(w_{11}^{XX} - w_{10}^{XX}) + w_{10}^{XX} - C_h) \\
&+(1-\theta_h)q(\theta_h(w_{01}^{XG} - w_{00}^{XG}) + w_{00}^{XG} - C_h) + (1-\theta_h)(1-q)(\theta_l(w_{01}^{XB} - w_{00}^{XB}) + w_{00}^{XB} - C_l)\} \\
&(1-\alpha)\{\theta_l(w_{11}^{XX} - w_{10}^{XX}) + w_{10}^{XX} - C_h) + (1-\theta_l)(\theta_l(w_{01}^{XB} - w_{00}^{XB}) + w_{00}^{XB} - C_l)\} - C_h \\
&\geq \theta_l(\theta_l(w_{11}^{XX} - w_{10}^{XX}) + w_{10}^{XX} - C_h) + (1-\theta_l)(\theta_l(w_{01}^{XB} - w_{00}^{XB}) + w_{00}^{XB} - C_l) - C_l
\end{aligned} \tag{A.12}$$

I verify in the next step, Constraint A.12 is redundant. Constraint A.13 is the principal's truth-telling constraint.

$$w_{00}^{XG} - w_{00}^{XB} \geq \Delta \tag{A.13}$$

Constraint A.14 is the mimicking constraint of the principal in the bad market. One could verify that in parameter ranges in which PPS is the equilibrium, A.14 will be automatically satisfied.

$$\theta_l(\theta_l(2 - w_{11}^{XX}) + (1-\theta_l)(1 - w_{10}^{XX})) + (1-\theta_l)(\theta_l(1 - w_{01}^{XB}) + (1-\theta_l)(0 - w_{00}^{XB})) \\ \geq \theta_l(\theta_l(2 - \frac{C_l}{\theta_l}) + (1-\theta_l)(1 - \frac{C_l}{\theta_l})) + (1-\theta_l)(\theta_l(1 - \frac{C_l}{\theta_l}) + (1-\theta_l)(0-0)) \tag{A.14}$$

To solve the above maximization problem, I get the following contract:

$$w_{00}^{XB} = 0, w_{01}^{XB} = \frac{C_l}{\theta_l}, w_{00}^{XG} = \Delta, w_{01}^{XG} = \frac{C_h}{\theta_h} + \Delta, w_{11}^{XX} = \frac{C_h}{\bar{\theta}_\alpha} + w_{10}^{XX}$$
$$w_{10}^{XX} = \frac{1}{\theta_\alpha}\{(\theta_\alpha + 1)C_h + q\alpha\theta_h\Delta - (\alpha\theta_h\bar{\theta} + (1-\alpha)\theta_l^2)\frac{C_h}{\bar{\theta}_\alpha}\}$$

Verify that if the principal offers a contract which induces the agent to take the market-insensitive strategy under pooling, the principal will deviate and offer a different contract which induces the agent to take the market-sensitive strategy. In other words, there exists a contract which induces pooling at the market-sensitive strategy which defeats the contract inducing pooling at the market-insensitive strategy.

Following the argument in the previous step of SSP, one could easily verify that pooling at market-insensitive strategy in the second period is not renegotiation proof. The principal in the good market will always want to induce the agent to undertake the market-sensitive strategy. I next verify the principal in the good market will not pool at the market-insensitive strategy either in the first period. Assume that the principal offers w_{11}^{GX}, w_{10}^{GX}, w_{01}^{GG}, w_{00}^{GG}, w_{01}^{GB} and w_{00}^{GB}. To prevent the principal in the bad market from mimicking, the following equation must hold:

$$\theta_l(\theta_l(2 - w_{11}^{GX}) + (1-\theta_l)(1 - w_{10}^{GX})) + (1-\theta_l)(\theta_l(1 - w_{01}^{GB}) + (1-\theta_l)(0 - w_{00}^{GB}) \\ \leq \theta_l(\theta_l(2 - w_{11}^{XX}) + (1-\theta_l)(1 - w_{10}^{XX})) + (1-\theta_l)(\theta_l(1 - w_{01}^{XB}) + (1-\theta_l)(0 - w_{00}^{XB}) \tag{A.15}$$

If Constraint A.16 and A.17 hold, then Constraint A.23 holds.

$$\theta_l(2 - w_{11}^{GX}) + (1-\theta_l)(1 - w_{10}^{GX})) \leq \theta_l(2 - w_{11}^{XX}) + (1-\theta_l)(1 - w_{10}^{XX}) \tag{A.16}$$
$$\theta_l(1 - w_{01}^{GB}) + (1-\theta_l)(0 - w_{00}^{GB}) \leq \theta_l(1 - w_{01}^{XB}) + (1-\theta_l)(0 - w_{00}^{XB}) \tag{A.17}$$

Constraint A.16 and A.17 imply that $w_{10}^{GX} \geq w_{10}^{XX} + \Delta$ and $w_{00}^{GB} \geq w_{00}^{XB} + \Delta$. If the principal in the good market deviates, the change in profit is:

$$
\begin{aligned}
&\pi^D - \pi^{ND} \\
=&\theta_h\{q(\theta_h(2-w_{11}^{GX}) + (1-\theta_h)(1-w_{10}^{GX})) + (1-q)(\theta_l(2-w_{11}^{GX}) + (1-\theta_l)(1-w_{10}^{GX}))\} \\
+&(1-\theta_h)\{q(\theta_h(1-w_{01}^{GG}) + (1-\theta_h)(0-w_{00}^{GG})) + (1-q)(\theta_l(1-w_{01}^{GB}) + (1-\theta_l)(0-w_{00}^{GB}))\} \\
-&\theta_l(\theta_l(2-w_{11}^{XX}) + (1-\theta_l)(1-w_{10}^{XX})) - (1-\theta_l)(\theta_l(1-w_{01}^{XB}) + (1-\theta_l)(0-w_{00}^{XB})) \\
\geq&\theta_h\{q(\theta_h(2-w_{11}^{GX}) + (1-\theta_h)(1-w_{10}^{GX})) + (1-q)(\theta_l(2-w_{11}^{GX}) + (1-\theta_l)(1-w_{10}^{GX}))\} \\
+&(1-\theta_h)\{q(\theta_h(1-w_{01}^{GG}) + (1-\theta_h)(0-w_{00}^{GG})) + (1-q)(\theta_l(1-w_{01}^{GB}) + (1-\theta_l)(0-w_{00}^{GB}))\} \\
-&\theta_h(\theta_l(2-w_{11}^{XX}) + (1-\theta_l)(1-w_{10}^{XX})) - (1-\theta_h)(\theta_l(1-w_{01}^{XB}) + (1-\theta_l)(0-w_{00}^{XB}))
\end{aligned}
$$

Substitute w_{10}^{GX} and w_{10}^{GX} into the above equation,

$$\pi^D - \pi^{ND} \geq q(\theta_h - \theta_l)(1 - \frac{C_h}{\theta_h}) + \theta_h(1-q)\Delta \geq 0$$

Based on the concept of Undefeated Equilibrium, if the principal in the bad market does not follow, she will be considered as the bad type and offer a contract that fully separates himself. This contract corresponds to an annual bonus of $\frac{C_l}{\theta_l}$ and zero salary. The principal gets no less profit from this contract than the pooling contract inducing market-insensitive strategy, and is thus better off than (w_{10}^{GX}, w_{11}^{GX}, w_{01}^{GB}, w_{00}^{GB}).

Step 3 Proof of contract under other information revelation structures – SPS. Characterize the contract under SPS.

$$
\begin{aligned}
&\max_{w\{..\}} \; \theta_h\{q(\theta_h(2-w_{11}^{GX}) + (1-\theta_h)(1-w_{10}^{GX})) + (1-q)(\theta_l(2-w_{11}^{GX}) + (1-\theta_l)(1-w_{10}^{GX}))\} \\
+&(1-\theta_h)\{q(\theta_h(1-w_{01}^{GG}) + (1-\theta_h)(0-w_{00}^{GG})) + (1-q)(\theta_l(1-w_{01}^{GB}) + (1-\theta_l)(0-w_{00}^{GB}))\}
\end{aligned}
$$

To save space, limited liability constraints are not listed. Constraint A.18-A.21 are the agent's incentive constraints.

$$s.t. \;\bar{\theta}(w_{11}^{GX} - w_{10}^{GX}) \geq C_h \tag{A.18}$$

$$\theta_h(w_{01}^{GG} - w_{00}^{GG}) \geq C_h \tag{A.19}$$

$$\theta_l(w_{01}^{GB} - w_{00}^{GB}) \geq C_l \tag{A.20}$$

$$
\begin{aligned}
&\theta_h\{\bar{\theta}(w_{11}^{GX} - w_{10}^{GX}) + w_{10}^{GX} - C_h)\} + (1-\theta_h)\{q(\theta_h(w_{01}^{GG} - w_{00}^{GG}) + w_{00}^{GG} - C_h) \\
&\quad +(1-q)(\theta_l(w_{01}^{GB} - w_{00}^{GB}) + w_{00}^{GB} - C_l)\} - C_h \geq q(\theta_h(w_{01}^{GG} - w_{00}^{GG}) + w_{00}^{GG} - C_h) \\
&\quad\quad +(1-q)(\theta_l(w_{01}^{GB} - w_{00}^{GB}) + w_{00}^{GB} - C_l)
\end{aligned} \tag{A.21}
$$

Constraint A.22 and A.23 are the principal's truth-telling constraints.

$$w_{00}^{GG} \geq w_{00}^{GB} + \Delta \tag{A.22}$$

$$\theta_l^2 w_{11}^{GX} + \theta_l(1-\theta_l)w_{10}^{GX} + \theta_l(1-\theta_l)(1-w_{01}^{GB}) + (1-\theta_l)^2 w_{00}^{GB} \geq 2C_l \tag{A.23}$$

To solve the above maximization problem, I get the following contract:

$$w_{00}^{GB} = (1-q\theta_l)\Delta, w_{01}^{GB} = \frac{C_l}{\theta_l} + w_{00}^{GB}, w_{00}^{GG} = w_{00}^{GB} + \Delta$$
$$w_{01}^{GG} = \frac{C_h}{\theta_h} + \Delta + w_{00}^{GB}, w_{10}^{GX} = \frac{C_h}{\theta_h} + q\Delta + w_{00}^{GB}, w_{11}^{GX} = \frac{C_h}{\bar{\theta}} + \frac{C_h}{\theta_h} + q\Delta + w_{00}^{GB}$$

As in the derivation of SSP and PPS, one could easily verify that if the principal offers a contract which induces the agent to take the market-insensitive strategy under pooling, the principal will deviate and offer a different contract which induces the agent to take the market-sensitive strategy.

Prove that the principal in the good market is strictly better off in the SSS than in SPS.

$$\begin{aligned}
\Delta\pi =& \mathbb{E}\left[\pi^{SSS}\right] - \mathbb{E}\left[\pi^{SPS}\right] \\
=& \theta_h\{q(\theta_h(\frac{C_h}{\theta_h} + w_{00}^{GB} + \Delta - \frac{C_h}{\theta_h} - 2\Delta) + (1-\theta_h)(w_{00}^{GB} + \Delta - 2\Delta)) \\
&+(1-q)(\theta_l(\frac{C_l}{\theta_l} + w_{00}^{GB} - \frac{C_l}{\theta_l} - \Delta) + (1-\theta_l)(w_{00}^{GB} - \Delta))\} \\
&+(1-\theta_h)\{q(\theta_h(\frac{C_h}{\bar{\theta}} + w_{00}^{GB} + q\Delta - \frac{C_h}{\theta_h} - 2\Delta) + (1-\theta_h)(w_{00}^{GB} + q\Delta - 2\Delta)) \\
&+(1-q)(\theta_l(\frac{C_l}{\bar{\theta}} + w_{00}^{GB} + q\Delta - \frac{C_l}{\theta_l} - \Delta) + (1-\theta_l)(w_{00}^{GB} + q\Delta - \Delta))\} \\
=& -q\theta_l\Delta + \theta_l(C_l - \frac{\theta_l}{\bar{\theta}}C_h) + \theta_h C_h - \theta_h(qC_h + (1-q)C_l)
\end{aligned}$$

I then verify the monotonicity of $\Delta\pi$ over $q \in [0,1]$.

$$\begin{aligned}
\frac{\partial\Delta\pi}{\partial q} &= -\theta_l\Delta + \theta_h(C_h - C_l) + \frac{\theta_l^2}{\bar{\theta}^2}C_h(\theta_h - \theta_l) \\
(\text{let } q \to 0) &\to -\theta_l\Delta + \theta_h(C_h - C_l) + C_h(\theta_h - \theta_l) \\
&= (\theta_h - \theta_l)\Delta \\
&> 0
\end{aligned}$$

In aBBition, at $q = 0$ and $q = 1$ $\Delta\pi$ is non-negative:

$$\Delta\pi(q = 0) = (\theta_h - \theta_l)(C_h - C_l) > 0$$
$$\Delta\pi(q = 1) = -\theta_l\Delta + \theta_l\Delta + \theta_h C_h - \theta_h C_h = 0$$

Then $\Delta\pi$ is increasing over $q \in [0, 1]$. In other words, the principal in the good market is strictly better off in the SSS than in SPS. Following the proof here, one could verify that SPP will be dominated by SSP because the principal in the good market will not want to pool following good performance.

Step 4 Proof of contract under other information revelation structures – PSS. Characterize the contract under PSS.

$$\max_{w\{..\}} \theta_h\{q(\theta_h(2 - w_{11}^{XG}) + (1 - \theta_h)(1 - w_{10}^{XG})) + (1 - q)(\theta_l(2 - w_{11}^{XB}) + (1 - \theta_l)(1 - w_{10}^{XB}))\}$$
$$+(1 - \theta_h)\{q(\theta_h(1 - w_{01}^{XG}) + (1 - \theta_h)(0 - w_{00}^{XG})) + (1 - q)(\theta_l(1 - w_{01}^{XB}) + (1 - \theta_l)(0 - w_{00}^{XB}))\}$$

To save space, limited liability constraints are not listed. Constraint A.24-A.28 are the agent's incentive constraints, Constraint A.29 is the agent's project choice constraint.

$$s.t.\ \theta_h(w_{11}^{XG} - w_{10}^{XG}) \geq C_h \tag{A.24}$$
$$\theta_h(w_{01}^{XG} - w_{00}^{XG}) \geq C_h \tag{A.25}$$
$$\theta_l(w_{11}^{XB} - w_{10}^{XB}) \geq C_l \tag{A.26}$$
$$\theta_l(w_{01}^{XB} - w_{00}^{XB}) \geq C_l \tag{A.27}$$

$$\begin{aligned}
&\alpha\{q(\theta_h^2 w_{11}^{XG} + \theta_h(1 - \theta_h)(w_{10}^{XG} + w_{01}^{XG}) + (1 - \theta_h)^2 w_{00}^{XG} - C_h) + (1 - q) \\
&\quad(\theta_h\theta_l w_{11}^{XB} + \theta_h(1 - \theta_l)w_{10}^{XB} + \theta_l(1 - \theta_h)w_{01}^{XB} + (1 - \theta_h)(1 - \theta_l)w_{00}^{XB}) - C_l)\} \\
&+(1 - \alpha)\{\theta_l^2 w_{11}^{XB} + \theta_l(1 - \theta_l)(w_{10}^{XB} + w_{01}^{XB}) + (1 - \theta_l)^2 w_{00}^{XB} - C_l\} - C_h \\
&\quad\geq \alpha\{q(\theta_h w_{01}^{XG} + (1 - \theta_h)w_{00}^{XG} - C_h) + (1 - q)(\theta_l w_{01}^{XB} + (1 - \theta_l)w_{00}^{XB} \\
&\quad\; -C_l)\} + (1 - \alpha)(\theta_l w_{01}^{XB} + (1 - \theta_l)w_{00}^{XB} - C_l)
\end{aligned} \tag{A.28}$$

$$\begin{aligned}
&\alpha\{q(\theta_h^2 w_{11}^{XG} + \theta_h(1 - \theta_h)(w_{10}^{XG} + w_{01}^{XG}) + (1 - \theta_h)^2 w_{00}^{XG} - C_h) + (1 - q) \\
&\quad(\theta_h\theta_l w_{11}^{XB} + \theta_h(1 - \theta_l)w_{10}^{XB} + \theta_l(1 - \theta_h)w_{01}^{XB} + (1 - \theta_h)(1 - \theta_l)w_{00}^{XB} - C_l)\} \\
&+(1 - \alpha)\{\theta_l^2 w_{11}^{XB} + \theta_l(1 - \theta_l)(w_{10}^{XB} + w_{01}^{XB}) + (1 - \theta_l)^2 w_{00}^{XB} - C_l\} - C_h \\
&\quad\geq \theta_l^2 w_{11}^{XB} + \theta_l(1 - \theta_l)(w_{10}^{XB} + w_{01}^{XB}) + (1 - \theta_l)^2 w_{00}^{XB} - 2C_l
\end{aligned} \tag{A.29}$$

Constraint A.30 and A.31 are the principal's truth-telling constraints.

$$w_{10}^{XG} - w_{10}^{XB} \geq \Delta \tag{A.30}$$
$$w_{00}^{XG} - w_{00}^{XB} \geq \Delta \tag{A.31}$$

If $\theta_\alpha \geq \frac{C_h}{C_l}\theta_l$, the principal in the bad market will want to mimic. Rearrange $A.27$ and $A.28$:

$$(A.27) \rightarrow (A.27') \ w_{10}^{XB} - w_{00}^{XB} \geq \frac{C_h}{\theta_\alpha}$$
$$(A.28) \rightarrow (A.28') \ w_{10}^{XB} - w_{00}^{XB} \geq \frac{C_h - C_l - q\alpha\Delta}{\theta_\alpha - \theta_l} \frac{C_h - C_l - q\alpha\Delta}{\theta_\alpha - \theta_l} - \frac{C_h}{\theta_\alpha}$$
$$= \frac{\theta_l C_h - \theta_\alpha(C_l + q\alpha\Delta)}{(\theta_\alpha - \theta_l)\theta_\alpha}$$

As in the previous proof, one could easily verify that pooling at the market-insensitive strategy is not an equilibrium. As a result, Constraint $A.29$ will be redundant.

To solve the above maximization problem, I get the following contract if $\theta_\alpha \geq \frac{C_h}{C_l}\theta_l$:

$$w_{00}^{XB} = 0, w_{10}^{XB} = \frac{C_h}{\theta_\alpha}, w_{01}^{XB} = \frac{C_l}{\theta_l}, w_{11}^{XB} = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_\alpha};$$
$$w_{00}^{XG} = \Delta, w_{01}^{XG} = \frac{C_h}{\theta_h} + \Delta, w_{10}^{XG} = \frac{C_h}{\theta_\alpha} + \Delta, w_{11}^{XG} = \frac{C_h}{\theta_\alpha} + \frac{C_h}{\theta_h} + \Delta.$$

Step 4 Proof of contract under other information revelation structures – PSP dominated by PSS.

If the principal does not reveal the market condition in the first period, the agent will have to infer it from past performance following bad performance. Following good performance, the principal reveals the market condition, thus the agent does not have to infer it on her own.

$$\alpha' = Pr(\theta_1 = \theta_h | y_1 = 0) = \frac{\alpha(1-\theta_h)}{\alpha(1-\theta_h) + (1-\alpha)(1-\theta_l)} < \alpha \tag{A.32}$$

Define $\bar{\theta}_\alpha = \alpha' q\theta_h + (1 - \alpha' q)\theta_l$. From Lemma 2, if the agent knows information, she takes the adaptive strategy which fits the external market. If the agent does not know the information, I first characterize the contract under which the agent takes the market-sensitive strategy under pooling.

$$\max_{w\{..\}} \ \theta_h\{q(\theta_h(2 - w_{11}^{XG}) + (1-\theta_h)(1 - w_{10}^{XG})) + (1-q)(\theta_l(2 - w_{11}^{XB}) + (1-\theta_l)(1 - w_{10}^{XB}))\}$$
$$+(1-\theta_h)\{q(\theta_h(1 - w_{01}^{XX}) + (1-\theta_h)(0 - w_{00}^{XX})) + (1-q)(\theta_l(1 - w_{01}^{XX}) + (1-\theta_l)(0 - w_{00}^{XX}))\}$$

To save space, limited liability constraints are not listed. Constraint A.33-A.36 are the agent's incentive constraints, constraint A.37 is the agent's project choice constraint.

$$s.t.\ \bar{\theta}_\alpha(w_{11}^{XG} - w_{10}^{XG}) \geq C_h \tag{A.33}$$
$$\theta_l(w_{11}^{XB} - w_{10}^{XB}) \geq C_l \tag{A.34}$$
$$\bar{\theta}_\alpha(w_{01}^{XX} - w_{00}^{XX}) \geq C_h \tag{A.35}$$

$$\begin{aligned}
&\alpha\{\theta_h q(\theta_h(w_{11}^{XG} - w_{10}^{XG}) + w_{10}^{XG} - C_h) + (1-q)\theta_h(\theta_l(w_{11}^{XB} - w_{10}^{XB}) + w_{10}^{XB} - C_l) \\
&+(1-\theta_h)q(\theta_h(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h) + (1-\theta_h)(1-q)(\theta_l(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h)\} \\
&(1-\alpha)\{\theta_l(\theta_l(w_{11}^{XB} - w_{10}^{XB}) + w_{10}^{XB} - C_l) + (1-\theta_l)(\theta_l(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h)\} - C_h \\
&\geq \alpha q(\theta_h(w_{01}^{XX} - w_{10}^{XX}) + w_{00}^{XX} - C_h) + (1-\alpha q)(\theta_l(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h)
\end{aligned} \tag{A.36}$$

$$\begin{aligned}
&\alpha\{\theta_h q(\theta_h(w_{11}^{XG} - w_{10}^{XG}) + w_{10}^{XG} - C_h) + (1-q)\theta_h(\theta_l(w_{11}^{XB} - w_{10}^{XB}) + w_{10}^{XB} - C_l) \\
&+(1-\theta_h)q(\theta_h(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h) + (1-\theta_h)(1-q)(\theta_l(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h)\} \\
&(1-\alpha)\{\theta_l(\theta_l(w_{11}^{XB} - w_{10}^{XB}) + w_{10}^{XB} - C_l) + (1-\theta_l)(\theta_l(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h)\} - C_h \\
&\geq \theta_l(\theta_l(w_{11}^{XB} - w_{10}^{XB}) + w_{10}^{XB} - C_l) + (1-\theta_l)(\theta_l(w_{01}^{XX} - w_{00}^{XX}) + w_{00}^{XX} - C_h) - C_l
\end{aligned} \tag{A.37}$$

Constraint A.38 is the principal's truth-telling constraint.

$$w_{10}^{XG} - w_{10}^{XB} \geq \Delta \tag{A.38}$$

Constraint A.39 is the mimicking constraint of the principal in the bad market.

$$\begin{aligned}
&\theta_l(\theta_l(2 - w_{11}^{XB}) + (1-\theta_l)(1 - w_{10}^{XB})) + (1-\theta_l)(\theta_l(1 - w_{01}^{XX}) + (1-\theta_l)(0 - w_{00}^{XX})) \\
&\geq \theta_l(\theta_l(2 - \frac{C_l}{\theta_l}) + (1-\theta_l)(1 - \frac{C_l}{\theta_l})) + (1-\theta_l)(\theta_l(1 - \frac{C_l}{\theta_l}) + (1-\theta_l)(0-0))
\end{aligned} \tag{A.39}$$

Rearrange A.36 and A.37, I get:

$$(A.36) \rightarrow (A.36') :$$
$$w_{10}^{XB} \geq \frac{1}{\theta_\alpha}\{C_h + \frac{C_h}{\bar{\theta}_\alpha}(q\alpha\theta_h + (1-q\alpha)\theta_l) - \alpha(1-\theta_h)\frac{\bar{\theta}}{\bar{\theta}_\alpha}C_h - (1-\alpha)(1-\theta_l)\frac{\theta_l}{\bar{\theta}_\alpha}C_h - q\alpha\theta_h\Delta\}$$
$$(A.37) \rightarrow (A.37') :$$
$$w_{10}^{XB} \geq \frac{1}{\theta_\alpha - \theta_l}\{(2-\theta_\alpha)C_h - (2-\theta_l)C_l + \alpha\frac{C_h}{\bar{\theta}_\alpha}((1-\theta_l)\theta_l - (1-\theta_h)\bar{\theta}) - q\alpha\theta_h\Delta\}$$

Deduct the left hand side of $A.36'$ from that of $A.37'$, I get:

Define $f(q) = A.37' - A.36'$

$$
\begin{aligned}
&= \frac{1}{\theta_\alpha - \theta_l}\{(2-\theta_\alpha)C_h - (2-\theta_l)C_l + \alpha\frac{C_h}{\bar{\theta}_\alpha}((1-\theta_l)\theta_l - (1-\theta_h)\bar{\theta}) - q\alpha\theta_h\Delta\} \\
&- \frac{1}{\theta_\alpha}\{C_h + \frac{C_h}{\bar{\theta}_\alpha}(q\alpha\theta_h + (1-q\alpha)\theta_l) - \alpha(1-\theta_h)\frac{\bar{\theta}}{\bar{\theta}_\alpha}C_h - (1-\alpha)(1-\theta_l)\frac{\theta_l}{\bar{\theta}_\alpha}C_h - q\alpha\theta_h\Delta\}
\end{aligned}
$$

One could easily verify that $f(q)$ is decreasing in q. $\exists\ \bar{q}$ such that if $q > \bar{q}$, A.36 binds. I later verify that the contract if A.36 binds will be strictly dominated by PSS, which means the contract if A.37 binds will also be strictly dominated by PSS. This is because A.37 is no longer a binding constraint if $q \leq \bar{q}$ and the contract subject to only A.36 offers the principal a higher payoff than A.37.

To solve the above maximization problem, I get the following contract:

$$
\begin{aligned}
w_{10}^{XB} &= \frac{1}{\theta_\alpha}\{C_h + \frac{C_h}{\bar{\theta}_\alpha}(q\alpha\theta_h + (1-q\alpha)\theta_l) - \alpha(1-\theta_h)\frac{\bar{\theta}}{\bar{\theta}_\alpha}C_h - (1-\alpha)(1-\theta_l)\frac{\theta_l}{\bar{\theta}_\alpha}C_h - q\alpha\theta_h\Delta\}, \\
w_{11}^{XB} &= \frac{C_l}{\theta_l} + w_{10}^{XB}, w_{10}^{XG} = w_{10}^{XB} + \Delta, w_{11}^{XG} = \frac{C_h}{\theta_h} + w_{10}^{XB} + \Delta.
\end{aligned}
$$

I then verify that the contract if A.36 binds will be strictly dominated by PSS.

$$
\begin{aligned}
\Delta\pi(q) =& \mathbb{E}\left[\pi^{PSS}\right] - \mathbb{E}\left[\pi^{PSP}\right] \\
=& \theta_h(w_{10}^{XB} - \frac{C_h}{\theta_\alpha}) + (1-\theta_h)(1-q)\theta_l(\frac{C_h}{\theta_\alpha} - \frac{C_l}{\theta_l}) + (1-\theta_h)q((\frac{C_h}{\bar{\theta}_\alpha} - \frac{C_h}{\theta_h})\theta_h - \Delta) \\
\Delta\pi(q=0) =& \frac{\theta_h}{\theta_\alpha}(\theta_l C_h + \alpha(\theta_h - \theta_l)C_h) + (1-\theta_h)(C_h - C_l) \\
>& 0
\end{aligned}
$$

Define the following $M(q)$ and $N(q)$:

$$
\begin{aligned}
M(q) =& (1-\theta_h)(1-q)\theta_l(\frac{C_h}{\theta_\alpha} - \frac{C_l}{\theta_l}) + (1-\theta_h)q((\frac{C_h}{\bar{\theta}_\alpha} - \frac{C_h}{\theta_h})\theta_h - \Delta) \\
N(q) =& \theta_h w_{10}^{XB}
\end{aligned}
$$

Take the derivative of $M(q)$ and $N(q)$ w.r.t. q:

$$
\begin{aligned}
\frac{dM(q)}{dq} =& (\theta_h - \theta_l)C_h(\underbrace{\frac{1}{\bar{\theta}_\alpha} - \frac{1-\theta_h}{\theta_h}}_{>0} + \frac{\bar{\theta}}{\bar{\theta}_\alpha^2}) \\
\frac{dN(q)}{dq} =& \frac{\alpha C_h\theta_h}{\theta_\alpha\bar{\theta}_\alpha}(\theta_h - \theta_l)\{\theta_h - \frac{\alpha'}{\alpha\bar{\theta}_\alpha}(\theta_l^2 - \alpha\theta_l^2 + \alpha\theta_l\theta_h + q\alpha\theta_h(\theta_h - \theta_l))\}
\end{aligned}
$$

One could verify that:

$$(\theta_h-\theta_l)C_h\frac{\bar{\theta}}{\bar{\theta}_\alpha^2}-\frac{\alpha C_h\theta_h}{\theta_\alpha\bar{\theta}_\alpha}(\theta_h-\theta_l)\frac{\alpha'}{\alpha\bar{\theta}_\alpha}(\theta_l^2-\alpha\theta_l^2+\alpha\theta_l\theta_h+q\alpha\theta_h(\theta_h-\theta_l))>0$$

Given that $\mathbb{E}\left[\pi^{PSS}\right]-\mathbb{E}\left[\pi^{PSP}\right]$ is increasing in q and $\Delta\pi(q=0)=0$, the principal in the good market will be better off in the PSS than in PSP. Following the proof here, one could verify that PPP will be dominated by PPS because the principal in the good market will not want to pool following bad performance

Step 5 Compare SSS to PPS, PSS and SSP.

Verify that the principal in the good market is better off in the PPS than in SSS if α is sufficiently small and q is sufficiently large.

$$\begin{aligned}\Delta\pi(\alpha,q)=&\mathbb{E}\left[\pi^{SSS}\right]-\mathbb{E}\left[\pi^{PPS}\right]\\ =&\theta_h(w_{10}^{XX}-\frac{C_h}{\theta_\alpha})+\theta_h\{q(\theta_h(\frac{C_h}{\bar{\theta}_\alpha}-\frac{C_h}{\theta_h})-2\Delta)+(1-q)(\theta_h(\frac{C_h}{\bar{\theta}_\alpha}-\frac{C_l}{\theta_l})-\Delta)\}\\ &+(1-\theta_h)\{q(\theta_h(\frac{C_h}{\theta_h}-\frac{C_h}{\theta_h})-\Delta)+(1-q)(\theta_h(\frac{C_h}{\bar{\theta}_\alpha}-\frac{C_l}{\theta_l})-\Delta)\}\end{aligned}$$

$$\Delta\pi(\alpha=0,q=1)=0-(1+\theta_h)(C_h+\Delta)+\frac{\theta_h^2}{\theta_l}C_h<0$$

As in the proof of PSP, one could easily show that $\mathbb{E}\left[\pi^{SSS}\right]-\mathbb{E}\left[\pi^{PPS}\right]$ is increasing in q and decreasing in α, the principal in the good market will be better off in the PPS than in SSS if α is sufficiently small and q is sufficiently large.

Verify that the principal in the good market is better off in the PSS than in SSS if α is sufficiently large.

$$\begin{aligned}\Delta\pi(\alpha)=&\mathbb{E}\left[\pi^{SSS}\right]-\mathbb{E}\left[\pi^{PSS}\right]\\ =&\theta_h(\frac{C_h}{\theta_\alpha}-\frac{C_h}{\theta_h})+q(-\theta_h\Delta-(1-\theta_h)\Delta)+(1-q)(-\theta_l\Delta-(1-\theta_l)\Delta)\\ =&\frac{\theta_h}{\theta_\alpha}C_h-C_h-\Delta\end{aligned}$$

$$\Delta\pi(\alpha=0)=\frac{\theta_h}{\theta_l}C_h-C_h-C_l+\frac{\theta_l}{\theta_h}C_h\geq C_h-C_l>0$$

$$\Delta\pi(\alpha=1)=-\Delta<0$$

One could easily show that $\mathbb{E}\left[\pi^{SSS}\right]-\mathbb{E}\left[\pi^{PPS}\right]$ is decreasing in α. The principal in the good market is better off in the PSS than in SSS if α is sufficiently large.

Verify that the principal in the good market is better off in the SSP than in SSS if

α is q is sufficiently small.

$$
\begin{aligned}
\Delta\pi(q) =&\mathbb{E}\left[\pi^{SSS}\right]-\mathbb{E}\left[\pi^{SSP}\right]\\
=&(1-\theta_h)\{q(\theta_h(\frac{C_h}{\bar{\theta}}+w_{00}^{GX}-\frac{C_h}{\theta_h}-2\Delta)+(1-\theta_h)(w_{00}^{GX}-2\Delta))\\
&+(1-q)(\theta_l(\frac{C_h}{\bar{\theta}}+w_{00}^{GX}-\frac{C_l}{\theta_l}-\Delta)+(1-\theta_l)(w_{00}^{GX}-\Delta))\}\\
&+\theta_h\{q(\theta_h(\frac{C_h}{\theta_h}+w_{00}^{GX}-q\Delta+\Delta-\frac{C_h}{\theta_h}-2\Delta)+(1-\theta_h)(w_{00}^{GX}-q\Delta+\Delta-2\Delta))\\
&+(1-q)(\theta_l(\frac{C_l}{\theta_l}+w_{00}^{GX}-q\Delta-\frac{C_l}{\theta_l}-\Delta)+(1-\theta_l)(w_{00}^{GX}-q\Delta-\Delta))\}\\
=&C_h-q(C_h-C_l)-(1-\theta_h)\frac{\theta_l}{\bar{\theta}}C_h-C_l\theta_l-(1-\theta_l)q\Delta-\theta_h(1-q)(C_h-C_l)
\end{aligned}
$$

Assume $G(q)=\bar{\theta}\Delta\pi(q)$. The value of $G(q)$ at $q=1$ and $q=0$ is:

$$
\begin{aligned}
G(q=1) =&0\\
G(q=0) =&-\theta_l(C_h-C_l)(\theta_h-\theta_l)<0
\end{aligned}
$$

I then check whether $G(q)$ is greater then zero or not.

$$
\begin{aligned}
G(q) =&aq^2+bq+c\\
a =&(\theta_h-\theta_l)^2(C_h-\frac{C_h}{\theta_h}-\Delta)<0\\
b =&(\theta_h-\theta_l)(C_h(1-\theta_h)+(\theta_h-\theta_l)C_l+\theta_l(C_h-C_l)-\frac{C_h}{\theta_h}\theta_l(1-\theta_l))\\
G'(q=1) =&2a+b=(\theta_h-\theta_l)(C_h\theta_h-C_h-\Delta\theta_h-\frac{C_h}{\theta_h}\theta_l(1+\theta_l))<0
\end{aligned}
$$

Based on the above inequalities, it is obvious that $\exists\ \bar{q}$, if $q<\bar{q}$ then $\mathbb{E}\left[\pi^{SSS}\right]-\mathbb{E}\left[\pi^{SSP}\right]<0$. The principal in the good market is better off in the SSP than in SSS if α is q is sufficiently small.

Step 6 If α is sufficiently small and q is sufficiently large, PSS is the equilibrium information structure.

$$
\begin{aligned}
\Delta\pi(q,\alpha) =&\mathbb{E}\left[\pi^{PSS}\right]-\mathbb{E}\left[\pi^{PPS}\right]\\
=&\theta_h(w_{10}^{XX}-\frac{C_h}{\theta_\alpha})+\theta_h q((\frac{C_h}{\bar{\theta}_\alpha}-\frac{C_h}{\theta_h})\theta_h-\Delta)+\theta_h(1-q)\theta_l(\frac{C_h}{\theta_\alpha}-\frac{C_l}{\theta_l})
\end{aligned}
$$

$$
\begin{aligned}
\Delta\pi(q=1,\alpha=1) =&0\\
\Delta\pi(q=1,\alpha=0) >&0
\end{aligned}
$$

Following the proof in PSP, I show that $\mathbb{E}\left[\pi^{PSS}\right] - \mathbb{E}\left[\pi^{PPS}\right]$ is increasing in q: Define the following $M(q)$ and $N(q)$:

$$\begin{aligned} M(q,\alpha) =& \theta_h q((\frac{C_h}{\bar{\theta}_\alpha} - \frac{C_h}{\theta_h})\theta_h - \Delta) + \theta_h(1-q)\theta_l(\frac{C_h}{\theta_\alpha} - \frac{C_l}{\theta_l}) \\ N(q,\alpha) =& \theta_h w_{10}^{XX} \end{aligned}$$

Take the derivative of $M(q)$ and $N(q)$ w.r.t. q:

$$\begin{aligned} \frac{\partial M(q,\alpha)}{\partial q} =& \frac{\theta_h}{1-\theta_h}(\theta_h - \theta_l)C_h(\underbrace{\frac{1}{\bar{\theta}_\alpha} - \frac{1-\theta_h}{\theta_h}}_{>0} + \frac{\bar{\theta}}{\bar{\theta}_\alpha^2}) \\ \frac{\partial N(q,\alpha)}{\partial q} =& \frac{\theta_h}{\theta_\alpha}\{\theta_h \alpha \Delta + C_h \frac{\theta_h - \theta_l}{\bar{\theta}_\alpha^2}(\alpha\theta_h(\alpha-1)\theta_l + (1-\alpha)\theta_l^2 \alpha')\} \end{aligned}$$

One could verify that:

$$\frac{\theta_h}{1-\theta_h}(\theta_h - \theta_l)C_h(\frac{\bar{\theta}}{\bar{\theta}_\alpha^2}) - \frac{\theta_h}{\theta_\alpha}C_h\frac{\theta_h - \theta_l}{\bar{\theta}_\alpha^2}\alpha\theta_h(\alpha-1)\theta_l > 0$$

One could also prove that

$$\begin{aligned} \frac{\partial M(q=1,\alpha=1)}{\partial \alpha} >& 0 \\ \frac{\partial M(q=1,\alpha=0)}{\partial \alpha} >& 0 \end{aligned}$$

Given that $\mathbb{E}\left[\pi^{PSS}\right] - \mathbb{E}\left[\pi^{PPS}\right]$ is increasing in q, $\Delta\pi(q=1,\alpha=1) = 0$ and $\Delta\pi(q=1,\alpha=0) > 0$, the principal in the good market will be better off in the PSS than in PPS if α is sufficiently small and q is sufficiently large. If Δ is sufficiently large, PSS is dominated by PPS, because:

$$\begin{aligned} \Delta\pi(q=0) =& -\frac{\theta_h}{\theta_\alpha}\Delta + \theta_h(C_h - C_l) \\ \leq& -\Delta + \theta_h(C_h - C_l) \end{aligned}$$

Step 7 If α and q are sufficiently small, adaptive innovation (SSP) is the equilibrium information structure. If α and q are sufficiently large, innovation inertia (PPS) is the equilibrium information structure.

$$\begin{aligned}\Delta\pi(\alpha,q)=&\mathbb{E}\left[\pi^{SSP}\right]-\mathbb{E}\left[\pi^{PPS}\right]\\=&\theta_h(w^{XX}_{10}-\frac{C_h}{\theta_\alpha})+\theta_h\{q(\theta_h(\frac{C_h}{\bar\theta_\alpha}-\frac{C_h}{\theta_h})-(w^{GX}_{00}+\Delta-q\Delta))+(1-q)\\&(\theta_l(\frac{C_h}{\bar\theta_\alpha}-\frac{C_l}{\theta_l})-(w^{GX}_{00}-q\Delta))\}+(1-\theta_h)\{q(\theta_h(\frac{C_h}{\theta_h}-\frac{C_h}{\bar\theta})+\Delta-w^{GX}_{00})\\&+(1-q)(\theta_l(\frac{C_l}{\theta_l}-\frac{C_h}{\bar\theta})-w^{GX}_{00})\}\\\Delta\pi(\alpha=0,q=1)=&-(1+\theta_h)(C_h+\frac{\theta_h^2}{\theta_l}C_h)<0\\\Delta\pi(\alpha=1,q=1)=&-C_h-\Delta<0\\\Delta\pi(\alpha=0,q=0)=&\theta_h(\frac{C_h}{\theta_l}-\frac{C_h}{\theta_h})-\Delta+(2\theta_h-\theta_l)(C_h-C_l)>0\end{aligned}$$

In aBBition, I show the following derivatives:

$$\begin{aligned}\frac{\partial\Delta\pi(\alpha=0,q=0)}{\partial q}>&0 &\qquad \frac{\partial\Delta\pi(\alpha=1,q=1)}{\partial q}>&0\\\frac{\partial\Delta\pi(\alpha=0,q=1)}{\partial q}>&0 &\qquad \frac{\partial\Delta\pi(\alpha=0,q=0)}{\partial q}>&0\\\frac{\partial\Delta\pi(\alpha=1,q=0)}{\partial \alpha}>&0 &\qquad \frac{\partial\Delta\pi(\alpha=0,q=1)}{\partial \alpha}>&0\\\frac{\partial\Delta\pi(\alpha=0,q=0)}{\partial \alpha}>&0 &\qquad \frac{\partial\Delta\pi(\alpha=1,q=1)}{\partial \alpha}>&0\end{aligned}$$

Therefore, there $\exists\ \bar q,\bar\alpha$ if $q\geq\bar q$ and $\alpha\geq\bar\alpha$,$\mathbb{E}\left[\pi^{SSP}\right]-\mathbb{E}\left[\pi^{PPS}\right]\leq 0$. There $\exists\ \underline{q},\underline{\alpha}$ if $q\leq\underline{q}$ and $\alpha\leq\underline{\alpha}$,$\mathbb{E}\left[\pi^{SSP}\right]-\mathbb{E}\left[\pi^{PPS}\right]\geq 0$. Q.E.D.

Proof of Lemma 3 and 4.

Proof 1. Characterize the contract of a least costly separating equilibrium.

$$\begin{aligned}\max_{w\{..\}}\quad&\theta_h(1-w^h_1)+(1-\theta_h)(-w^h_0)\\s.t.\quad&\theta_h w^h_1+(1-\theta_h)w^h_0-C_h\geq w^h_0\\&\theta_l(1-w^l_1)+(1-\theta_l)(-w^l_0)\geq\theta_l(1-w^h_1)+(1-\theta_l)(-w^h_0)\\&w^h_1,w^h_0\geq 0\end{aligned}$$

One could easily verify that $w^h_1=\frac{C_h}{\theta_h}$, $w^h_0=\Delta$, $w^l_1=\frac{C_l}{\theta_l}$, $w^l_0=0$.

Consider $\theta_\alpha\leq\frac{C_h\theta_h}{C_h+\Delta}$. In this case, the principal in the good market is better off in the least costly separating equilibrium than in the efficient pooling equilibrium.

$\pi_h^{separating} = \theta_h(1-\frac{C_h}{\theta_h}) - \Delta$, $\pi_h^{pooling,s} = \theta_h(1-\frac{C_h}{\theta_\alpha})$ if pooling at the market-sensitive strategy, and $\pi_h^{pooling,i} = \theta_l(1-\frac{C_l}{\theta_l})$ if pooling at the market-sensitive strategy

$$
\begin{aligned}
\pi_h^{separating} - \pi_h^{pooling,s} &= \frac{\theta_h}{\theta_\alpha}C_h - C_h - (C_l - \frac{\theta_l}{\theta_h}C_h) \\
&= (\frac{\theta_h}{\theta_\alpha} + \frac{\theta_l}{\theta_h})C_h - (C_h + C_l) \\
&\geq 0 \\
\pi_h^{separating} - \pi_h^{pooling,i} &= \theta_h - \theta_l - \Delta - (C_h - C_l) \\
&= (\theta_h - C_h)(1 - \frac{\theta_l}{\theta_h}) \\
&\geq 0
\end{aligned}
$$

In this case, the least costly separating equilibrium is interim efficient in that there is no other incentive compatible contract that manages to strictly improve the equilibrium payoff of the agent or of at least one type of the principal without causing another payoff to deteriorate. The principal in the good market strictly prefers this contract over any contract resulting in pooling or any contract with more costly separation.

2. $\theta_\alpha > \frac{C_h\theta_h}{C_h+\Delta}$. In this case, the principal in the good market is worse off in the least costly separating equilibrium than in the efficient pooling equilibrium. The least costly separating equilibrium can be improved upon in a Pareto sense.

Q.E.D.

Proof of Proposition 3.

Proof

$$
\begin{aligned}
\max_{w\{..\}} \quad & \theta_h\{q(\theta_h(2-w_{11}^{GG}) + (1-\theta_h)(1-w_{10}^{GG})) + (1-q)(\theta_l(2-w_{11}^{GB}) + (1-\theta_l)(1-w_{10}^{GB}))\} \\
& +(1-\theta_h)\{q(\theta_h(1-w_{01}^{GG}) + (1-\theta_h)(0-w_{00}^{GG})) + (1-q)(\theta_l(1-w_{01}^{GB}) + (1-\theta_l)(0-w_{00}^{GB}))\}
\end{aligned}
$$

To save space, limited liability constraints are not listed. Constraint A.40-A.44 are the agent's incentive constraints, and A.45 is the agent's project choice constraint.

$$s.t.\ \theta_h(w_{11}^{GG} - w_{10}^{GG}) \geq C_h \tag{A.40}$$
$$\theta_h(w_{01}^{GG} - w_{00}^{GG}) \geq C_h \tag{A.41}$$
$$\theta_l(w_{11}^{GB} - w_{10}^{GB}) \geq C_l \tag{A.42}$$
$$\theta_l(w_{01}^{GB} - w_{00}^{GB}) \geq C_l \tag{A.43}$$
$$\begin{aligned}&\theta_h\{q(\theta_h(w_{11}^{GG} - w_{10}^{GG}) + w_{10}^{GG} - C_h) + (1-q)(\theta_l(w_{11}^{GB} - w_{10}^{GB}) + w_{10}^{GB} - C_l)\} \\ &+(1-\theta_h)\{q(\theta_h(w_{01}^{GG} - w_{00}^{GG}) + w_{00}^{GG} - C_h) + (1-q)(\theta_l(w_{01}^{GB} - w_{00}^{GB}) + w_{00}^{GB} - C_l)\} \\ &-C_h \geq q(\theta_h(w_{01}^{GG} - w_{00}^{GG}) + w_{00}^{GG} - C_h) + (1-q)(\theta_l(w_{01}^{GB} - w_{00}^{GB}) + w_{00}^{GB} - C_l\end{aligned} \tag{A.44}$$

Constraint A.45-A.47 are the principal's truth-telling constraints.

$$\theta_l(2 - w_{11}^{GB}) + (1-\theta_l)(1 - w_{10}^{GB}) \geq \theta_l(2 - w_{11}^{GG}) + (1-\theta_l)(1 - w_{10}^{GG}) \tag{A.45}$$
$$\theta_l(1 - w_{01}^{GB}) + (1-\theta_l)(0 - w_{00}^{GB}) \geq \theta_l(1 - w_{01}^{GG}) + (1-\theta_l)(0 - w_{00}^{GG}) \tag{A.46}$$
$$\begin{aligned}&\theta_l^2(2 - w_{11}^{BB}) + \theta_l(1-\theta_l)(1 - w_{10}^{BB}) + \theta_l(1-\theta_l)(1 - w_{01}^{BB}) + (1-\theta_l)^2(0 - w_{00}^{BB}) \\ &\geq \theta_l^2(2 - w_{11}^{GB}) + \theta_l(1-\theta_l)(1 - w_{10}^{GB}) + \theta_l(1-\theta_l)(1 - w_{01}^{GB}) + (1-\theta_l)^2(0 - w_{00}^{GB})\end{aligned} \tag{A.47}$$

Interim incentive constraints of the agent guarantee that the contract is interim renegotiation-proof.

$$\begin{aligned}&(A.44) \to (A.44') \qquad q\theta_h(w_{10}^{GG} - w_{00}^{GG}) + (1-q)\theta_h(w_{10}^{GB} - w_{00}^{GB}) \geq C_h \\ &(A.45) \to (A.45') \qquad \theta_l(w_{11}^{GG} - w_{10}^{GG}) - \theta_l(w_{11}^{GB} - w_{10}^{GB}) \geq w_{10}^{GB} - w_{10}^{GG} \\ &(A.46) \to (A.46') \qquad \theta_l(w_{01}^{GG} - w_{00}^{GG}) - \theta_l(w_{01}^{GB} - w_{00}^{GB}) \geq w_{00}^{GB} - w_{00}^{GG} \\ &(A.47) \to (A.47') \qquad \theta_l^2 w_{11}^{GB} + \theta_l(1-\theta_l)(w_{10}^{GB} + w_{01}^{GB}) + (1-\theta_l)^2 w_{00}^{GB} \geq 2C_l\end{aligned}$$

Rearrange the above equations, I get:

$$\begin{aligned}(A.40), (A.45') \qquad & w_{10}^{GG} \geq w_{10}^{GB} + \Delta \\ (A.41), (A.43), (A.7') \qquad & w_{00}^{GG} \geq w_{00}^{GB} + \Delta \\ (A.44)', (A.45') \qquad & w_{10}^{GB} \geq w_{00}^{GB} + \frac{C_h}{\theta_h}\end{aligned}$$

Substitute all into $(A.47')$, one could solve for w_{00}^{GB}, and all other variables.

$$\begin{aligned}&w_{00}^{GB} = \Delta; \qquad w_{01}^{GB} = \frac{C_l}{\theta_l} + \Delta; \qquad w_{10}^{GB} = \frac{C_h}{\theta_h} + \Delta; \qquad w_{11}^{GB} = \frac{C_l}{\theta_l} + \frac{C_h}{\theta_h} + \Delta; or \\ &w_{00}^{GG} = 2\Delta; \qquad w_{01}^{GG} = \frac{C_l}{\theta_l} + 2\Delta; \qquad w_{10}^{GG} = \frac{C_h}{\theta_h} + 2\Delta; \qquad w_{11}^{GG} = 2\frac{C_h}{\theta_h} + 2\Delta.\end{aligned}$$

Q.E.D.

Proof of Corollary 2.

Proof 1. If $m_1 = G$ in the first period, because $w_{11}^{GB} - w_{01}^{GB} = w_{10}^{GB} - w_{00}^{GB} = w_{11}^{GG} - w_{01}^{GG} = w_{10}^{GG} - w_{00}^{GG} = \frac{C_h}{\theta_h}$, the first period incentive pay is $\frac{C_h}{\theta_h}$ if $y_1 = 1$.

2. If $m_2 = G$ in the second period, because $w_{11}^{GG} - w_{10}^{GG} = w_{01}^{GG} - w_{00}^{GG} = \frac{C_h}{\theta_h}$, the second period incentive pay is $\frac{C_h}{\theta_h}$ if $y_2 = 1$.

3. If $m_2 = B$ in the second period, because $w_{11}^{GB} - w_{10}^{GB} = w_{01}^{GB} - w_{00}^{GB} = \frac{C_l}{\theta_l}$, the first period incentive pay is $\frac{C_l}{\theta_l}$ if $y_2 = 1$.

4. Because $w_{11}^{GG} - w_{11}^{GB} = w_{10}^{GG} - w_{10}^{GB} = w_{01}^{GG} - w_{01}^{GB} = w_{00}^{GG} - w_{00}^{GB} = \Delta$, if $\theta_2 = \theta_h$, the second period salary should increase by Δ from the first period. Because the hl type has a salary Δ since $w_{00}^{GB} = \Delta$. I set first period salary to $\frac{1}{2}\Delta$ and second period to $\frac{1}{2}\Delta$ for hl and $\frac{3}{2}\Delta$ for hh.

Q.E.D.

Proof of Proposition 4 and 5.

Proof Please refer to the proof in Proposition 2. Q.E.D.

Proof of Corollary 3 and 4.

Proof The results can be easily obtained following the proof in Corollary 2. Q.E.D.

Proof of Lemma 5.

Proof

$$\max_{w\{..\}} \quad \theta_h\{q(\theta_h(2 - w_{11}^{GG}) + (1-\theta_h)(1 - w_{10}^{GG})) + (1-q)(\theta_l(2 - w_{11}^{GB}) + (1-\theta_l)(1 - w_{10}^{GB}))\}$$
$$+(1-\theta_h)\{q(\theta_h(1 - w_{01}^{GG}) + (1-\theta_h)(0 - w_{00}^{GG})) + (1-q)(\theta_l(1 - w_{01}^{GB}) + (1-\theta_l)(0 - w_{00}^{GB}))\}$$

To save space, limited liability constraints are not listed. Constraint A.48-A.52 are the agent's incentive constraints.

$$s.t. \; \theta_h(w_{11}^{GG} - w_{10}^{GG}) \geq C_h \tag{A.48}$$
$$\theta_h(w_{01}^{GG} - w_{00}^{GG}) \geq C_h \tag{A.49}$$
$$\theta_l(w_{11}^{GB} - w_{10}^{GB}) \geq C_l \tag{A.50}$$
$$\theta_l(w_{01}^{GB} - w_{00}^{GB}) \geq C_l \tag{A.51}$$
$$\theta_h\{q(\theta_h(w_{11}^{GG} - w_{10}^{GG}) + w_{10}^{GG} - C_h) + (1-q)(\theta_l(w_{11}^{GB} - w_{10}^{GB}) + w_{10}^{GB} - C_l)\} \tag{A.52}$$
$$+(1-\theta_h)\{q(\theta_h(w_{01}^{GG} - w_{00}^{GG}) + w_{00}^{GG} - C_h) + (1-q)(\theta_l(w_{01}^{GB} - w_{00}^{GB}) + w_{00}^{GB} - C_l)\}$$
$$-C_h \geq q(\theta_h(w_{01}^{GG} - w_{00}^{GG}) + w_{00}^{GG} - C_h) + (1-q)(\theta_l(w_{01}^{GB} - w_{00}^{GB}) + w_{00}^{GB} - C_l$$

Constraint A.53-A.55 are the principal's truth-telling constraints.

$$\theta_l(2-w_{11}^{GB})+(1-\theta_l)(1-w_{10}^{GB}) \geq \theta_l(2-w_{11}^{GG})+(1-\theta_l)(1-w_{10}^{GG}) \quad \text{(A.53)}$$

$$\theta_l(1-w_{01}^{GB})+(1-\theta_l)(0-w_{00}^{GB})(1+t) \geq \theta_l(1-w_{01}^{GG})+(1-\theta_l)(0-w_{00}^{GG})(1+t) \quad \text{(A.54)}$$

$$\theta_l^2(2-w_{11}^{BB})+\theta_l(1-\theta_l)(1-w_{10}^{BB})+\theta_l(1-\theta_l)(1-w_{01}^{BB})+(1-\theta_l)^2(0-w_{00}^{BB})(1+t) \quad \text{(A.55)}$$
$$\geq \theta_l^2(2-w_{11}^{GB})+\theta_l(1-\theta_l)(1-w_{10}^{GB})+\theta_l(1-\theta_l)(1-w_{01}^{GB})+(1-\theta_l)^2(0-w_{00}^{GB})(1+t)$$

Solve the program above, one could obtain:

1. Compensation plan one: $w_{00}^{GB}=\Delta/(1+t(1-\theta_l)^2)$, $w_{01}^{GB}=\frac{C_l}{\theta_l}+w_{00}^{GB}$, $w_{10}^{GB}=\frac{C_h}{\theta_h}+w_{00}^{GB}$, and $w_{11}^{GB}=\frac{C_l}{\theta_l}+w_{10}^{GB}$;
2. Compensation plan two: $w_{00}^{GG}=\Delta/(1+t(1-\theta_l)^2)+\Delta/(1+t(1-\theta_l))$, $w_{01}^{GG}=\frac{C_l}{\theta_l}+w_{00}^{GG}$, $w_{10}^{GG}=\frac{C_h}{\theta_h}+w_{00}^{GG}$, and $w_{11}^{GG}=\frac{C_h}{\theta_h}+w_{10}^{GG}$.

Q.E.D.

Proof of Lemma 6.

Proof

$$\max_{w\{..\}} \ \theta_h\{q(\theta_h(2-w_{11}^{GG})+(1-\theta_h)(1-w_{10}^{GG}))+(1-q)(\theta_l(2-w_{11}^{GB})+(1-\theta_l)(1-w_{10}^{GB}))\}$$
$$+(1-\theta_h)\{q(\theta_h(1-w_{01}^{GX})+(1-\theta_h)(0-w_{00}^{GX}))+(1-q)(\theta_l(1-w_{01}^{GX})+(1-\theta_l)(0-w_{00}^{GX}))\}$$

To save space, limited liability constraints are not listed. Constraints A.56-A.59 are the agent's incentive constraints.

$$s.t.\ \theta_h(w_{11}^{GG}-w_{10}^{GG}) \geq C_h \quad \text{(A.56)}$$

$$\theta_l(w_{11}^{GB}-w_{10}^{GB}) \geq C_l \quad \text{(A.57)}$$

$$\bar{\theta}(w_{01}^{GX}-w_{00}^{GX}) \geq C_h \quad \text{(A.58)}$$

$$\theta_h\{q(\theta_h(w_{11}^{GG}-w_{10}^{GG})+w_{10}^{GG}-C_h)+(1-q)(\theta_l(w_{11}^{GB}-w_{10}^{GB})+w_{10}^{GB}-C_l)\} \quad \text{(A.59)}$$
$$+(1-\theta_h)\{\bar{\theta}(w_{01}^{GX}-w_{00}^{GX})+w_{00}^{GX}-C_h)\}-C_h \geq \bar{\theta}(w_{01}^{GX}-w_{00}^{GX})+w_{00}^{GX}-C_h$$

Constraint A.60 and A.61 are the principal's truth-telling constraints.

$$\theta_l(2-w_{11}^{GB})+(1-\theta_l)(1-w_{10}^{GB}) \geq \theta_l(2-w_{11}^{GG})+(1-\theta_l)(1-w_{10}^{GG}) \quad \text{(A.60)}$$

$$\theta_l^2(2-w_{11}^{BB})+\theta_l(1-\theta_l)(1-w_{10}^{BB})+\theta_l(1-\theta_l)(1-w_{01}^{BB})+(1-\theta_l)^2(0-w_{00}^{BB})(1+t) \quad \text{(A.61)}$$
$$\geq \theta_l^2(2-w_{11}^{GB})+\theta_l(1-\theta_l)(1-w_{10}^{GB})+\theta_l(1-\theta_l)(1-w_{01}^{GX})+(1-\theta_l)^2(0-w_{00}^{GX})(1+t)$$

Solve the program above, one could obtain:

1. Compensation plan one: $w_{00}^{GB} = ((1-\theta_l)(C_l - C_h\theta_l/\bar{\theta}) + \Delta + \beta\theta_l\Delta)/(1+t(1-\theta_l)^2)$, $w_{10}^{GB} = \frac{C_h}{\theta_h} - \beta\Delta/(1+t(1-\theta_l)^2) + w_{00}^{GB}$, $w_{01}^{GB} = C_h/\bar{\theta} + w_{00}^{GB}$, and $w_{11}^{GB} = \frac{C_l}{\theta_l} + w_{10}^{GB}$;
2. Compensation plan two: $w_{00}^{GG} = ((1-\theta_l)(C_l - C_h\theta_l/\bar{\theta}) + \Delta + \beta\theta_l\Delta)/(1+t(1-\theta_l)^2)$, $w_{10}^{GG} = \frac{C_h}{\theta_h} + \Delta/(1+t(1-\theta_l)) - \beta\Delta/(1+t(1-\theta_l)^2) + w_{00}^{GG}$, $w_{01}^{GG} = C_h/\bar{\theta} + w_{00}^{GG}$, and $w_{11}^{GG} = \frac{C_h}{\theta_h} + w_{10}^{GG}$.

Q.E.D.

Proof of Proposition 6.

Proof In early adaptation, the principal in the good market cannot improve his profit by revealing good information to the agent, as the agent will not give up the overly-powered incentive package. The principal is exactly at the point of having no incentive to renegotiate. The optimal contract that induces early adaptation therefore is weakly renegotiation-proof. Although credibly revealing news will not increase the profit of the principal in either market condition, it improves the agent's utility. It is therefore not strongly renegotiation-proof.

If $\frac{\theta_h - \theta_l}{\theta_l}(1 - \frac{C_h}{\theta_l}) - \frac{C_h - C_l}{\theta_l} > 0$, one can show that the agent could not find a contract that prevents the principal in a constantly good market from choosing the contract that is chosen by the principal in a deteriorating market. Q.E.D.

Proof of Proposition 7.

Proof Based on the proofs of Lemma 5 and 6:

$$\begin{aligned}\Delta\pi(t) =& \mathbb{E}\left[\pi^{SSS}\right] - \mathbb{E}\left[\pi^{PSS}\right] \\ =& (1-\theta_h)(C_h - \bar{C}) + (1-\theta_h)q(w_{00}^{GX} - w_{00}^{GG}) + (1-\theta_h)(1-q)(w_{00}^{GX} - w_{00}^{GB}) \\ &+ \theta_h(w_{00}^{GX} - w_{00}^{GB})\end{aligned}$$

The above formular sets a lower boundary for t. Define $a = 1 + (1-\theta_l)^2 t$.

$$\Delta\pi(t) = (C_h - C_l)(1 - \theta_h - \frac{1-\theta_l}{a})$$

If $t \geq \frac{\theta_h - \theta_l}{(1-\theta_h)(1-\theta_l^2)}$, then $\Delta\pi \geq 0$. Q.E.D.