Explaining Downward-rigid CEO Compensation: An Information Asymmetry Perspective

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Abstract: CEO compensation rarely gets cut, and almost every component of it increased in early 2000. I consider a two-period contracting problem in which a board privately knows its CEO's matching quality with the firm that changes over time. The board faces a trade-off: Revealing good information makes the CEO work harder, but it is costly. To save the information revelation cost, the board commits to a back-loaded compensation plan that features only upward adjustments in fixed and performance-based pay. This paper also considers extensions in which CEOs have transferable skills and sheds light on bonus caps and compensation disclosure policies.

Key words: CEO compensation, informed principal, signaling, matching quality, bonus cap, compensation disclosure

JEL codes: G38, J24, J31, M12

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During the 1990s and early 2000s, the total compensation of chief executive officers (CEOs) rose rapidly. Two phenomena have drawn researchers' attention. First, compensation for US CEOs was relatively flat in the decades leading up to the 1990s but increased dramatically during the 1990s and early 2000s (Frydman and Jenter, 2010). This sharp rise occurred during the tech boom. Second, almost every component of executive compensation, including salary and incentive pay, went up (Shue and Townsend, 2017). Cash compensation was not adjusted downward to offset the dramatic rise in long-term incentive pay. While the extant literature (e.g., Harris and Holmstrom, 1982) could justify downward rigidity in total compensation, it did not fit well with the executive compensation data in which each component increased.

This paper blends a dynamic informed principal model with a standard moral hazard model to explain the changing patterns of CEO pay. The starting point of this paper is to observe that, especially during the tech boom, two characteristics plagued the design of executive compensation: moral hazard and information asymmetry. While moral hazard is extensively studied in the literature, the combination of these two problems has been under-explored. Investigating the combined effect is crucial to understanding the trends of CEO compensation during the tech boom, as this period was marked by substantial high-tech investment and disruptive technological progress that increased the uncertainty in the market. Thus, apart from objective performance metrics, firms may have increasingly relied on private information to assess the (mis)match between the CEO's skills and the needs of the firm during that period.

The basic setup of the model is as follows. The board (the principal, she) has private information about the matching quality between the firm and the CEO (the agent, he), which can be either high or low. The principal's private information arrives sequentially at the beginning of each period over two periods. The agent is risk-neutral and needs to exert private effort in order to produce some output in each period. The matching quality is complementary to the agent's effort. In this paper, the principal can offer the agent a contract in order to motivate him to work. The general insight to be drawn from the model is that information asymmetry could be key to explaining recent patterns in CEO compensation. In addition to the traditional role of providing incentives, compensation plays another role: transferring information from the principal to the agent. In other words, the compensation structure serves as a means for the principal to convey her private information to the agent.

Several studies show that the board acquires and possesses private information about CEO performance. For example, Cornelli, Kominek, and Ljungqvist (2013) find that the board collects soft information to evaluate whether the CEO is a good match for the firm and the board makes firing decisions based on that information. A structural estimation by Taylor (2010) further indicates that the board's private information has more than five times greater influence on the board's beliefs in evaluating CEOs, compared to the public profitability signal. According to some recent surveys (Casal and Caspar, 2014; Larcker, Saslow, and Tayan, 2014), directors are well aware of CEOs' strengths and weaknesses and possess a great deal of expertise in analyzing the business environment due to their experiences serving on multiple boards in various industries.[1] Thus, as Hermalin (1998) points out, leaders face a temptation to mislead the less-informed due to their superior information; they must sacrifice or set an example (a costly action) in order to credibly signal their private information.

Because of the temptation to mislead the uninformed, the principal in this model faces a trade-off. Revealing high matching quality to the agent makes him work harder, as he would realize that his productivity is higher than he initially perceived it to be. The principal, however, needs to incur a cost of information revelation in order to

[1] According to a McKinsey report by Casal and Caspar (2014), the right directors are knowledgeable about their roles and able to commit sufficient time to analyzing what drives value. They also actively engage in strategic planning and look for potential development areas. "Boards need to look further out than anyone else in the company." The chairman of a leading energy company commented, "There are times when CEOs are the last ones to see changes coming."In a survey by Larcker, Saslow, and Tayan (2014), over half (55.1%) of directors report understanding the strengths and weaknesses of senior executives "extremely well" or "very well"; a third (33.5%) understand these strengths and weaknesses "moderately well"; and the remainder (11.4%) understand them "slightly well" or "not at all well."

convince the agent. In a dynamic setting, to save the information revelation cost, the principal with good information commits to a back-loaded compensation plan that consists of non-decreasing salary and non-decreasing performance-based pay. In other words, the compensation structure is used to provide signals to the agent.

I first analyze four benchmark cases: a one-period model with information asymmetry; a one-period model without information asymmetry; a two-period model without information asymmetry; and a two-period model with information asymmetry but no commitment. To facilitate the illustration in the introduction, I explain the equilibrium compensation structure only when offering equity pay is not allowed. Nevertheless, the main mechanism is robust to equity pay, as one can see in Section 3.3 that allows for equity pay.

In the two one-period benchmark cases, I find that if the production technology satisfies a certain condition that regulates the complementarity between the matching quality and the effort, the principal relies fully on fixed pay (or salary) to credibly communicate her private information to the agent. Performance-based (bonus) pay is set at a level as if the agent knew the matching quality. In other words, the bonus offered under information asymmetry is the same as the bonus offered under information symmetry. This is because, under this condition, signaling via bonus pay would involve sharing too much profit with the agent, thus outweighing the cost of signaling via salary. The principal, therefore, does not use bonus pay to signal her private information, but only to provide incentives. If the production technology does not satisfy the condition, the principal also uses bonus pay to provide signals, leading to either over- or under-provision of effort compared to the effort level under symmetric information.

The paper then considers a two-period model in which the matching quality may deteriorate in the second period. I choose a production technology that satisfies the aforementioned condition so that one can assign the signaling role to the salary and the incentive role to the bonus in a one-period model. Interestingly, the result that the bonus does not provide private information does not hold in a two-period setting. In other words, this technology allows me to attribute any increase in bonus pay in a two-period model over the amount paid in a one-period model to the signaling role, thus greatly simplifying the interpretation of my analysis.

Specifically, in the third benchmark case, in which information is symmetric in a two-period model, no salary is paid, as signaling is not needed. And bonus is the same as in the one-period model. I then consider the fourth benchmark case, in which information is asymmetric, but commitment is impossible. In this case, the principal does pay salaries to provide signals. The equilibrium contract, however, is stationary, in the sense that second-period compensation does not depend on first-period private information. It is a combination of two independent one-period contracts, which correspond to those offered under information asymmetry in a one-period model. Because the principal cannot commit to long-term contracts, the agent knows that the principal will make a new take-it-or-leave-it offer when new information arrives. Anticipating this, the agent will not agree to a package consisting of a higher bonus in the future and a lower salary today.

I then allow for commitment in the main analysis. The equilibrium contract exhibits downward rigidity in both salary and performance-based pay. The principal with high matching quality in the first period commits to a compensation schedule that consists of two compensation units. If the matching quality remains high in the second period, the principal chooses the unit that provides increasing (or at least not less) salaries over the two periods. This compensation structure provides the strongest signal, as salary does not encourage effort directly. Therefore, it is very costly for principals either with declining matching quality or constantly low matching quality to offer. If the matching quality declines, the principal chooses the unit that provides greater incentive pay in the second period. By increasing the second-period bonus, she saves the signaling cost in the form of salary that has neither signaling nor incentive value if the matching quality deteriorates. Because, with this compensation structure, the principal has to share more profit with the agent, it is more costly for a principal with constantly low matching quality to offer. If the matching quality is low in both periods, incentive pay that the principal offers in each period is the same and corresponds to the level in the one-period model.

My model generates three main empirical predictions. First, downwardly rigid contracts are more likely to be observed when there is greater uncertainty about the matching quality between firms and CEOs. As mentioned earlier, CEO compensation increased dramatically during the 1990s and early 2000s (Frydman and Jenter, 2010). This sharp rise occurred during the tech boom, a period with increased uncertainty due to substantial high-tech investment and disruptive technological progress. The drastic change in the market environment may have outdated certain types of managerial skills and accentuated the signaling role of compensation structure to retain CEOs with up-to-date skills. Second, downwardly rigid contracts are more likely to be observed in positions for which the performance assessment relies more on boards' subjective evaluation, such as R&D-oriented jobs and leadership positions. The purpose of offering such contracts is different from providing innovation incentives in Manso (2011), as compensation structure in my model is intended to provide signals. Third, depending on the production function, salary increases may send out a stronger signal of good subsequent matching quality (or performance) than do increases in performance-based pay.[2]

Internet technological innovations have greatly expanded product markets and imposed new challenges on managers working in a more diverse environment (Murphy and Zábojník, 2007). I consider an extension in which the agent possesses transferable skills, in the sense that better matching quality can bring greater outside option value to the agent. My model finds that the principal would offer a higher bonus instead of a higher salary to retain an agent who possesses more transferable skills. However, if the

[2] In practice, salary raises are one way for boards to give CEOs a sizable increase in total compensation without institutional constraints such as shareholder approval (for equity issuances to managers) or deviations from company-wide bonus plans. This is because bonuses and pensions are usually calculated in multiples of the salary (De Angelis and Grinstein, 2014; Bebchuk and Jackson, 2005).

agent's outside option value exceeds a certain level, the principal would not provide a signal, as attracting such an agent would imply too high a signaling cost.

This extension also sheds light on disclosure polices related to executive pay. Shue and Townsend (2017) find that a 2006 regulatory change that required firms to report the grant date fair value of option awards, not just the number of options, led to a significant decrease in the prevalence of number-rigid option grants. Although a sophisticated agent should have been able to infer the value from the number and the grant date share price, mandated disclosure of value likely had an impact on CEOs' outside option value in the labor market, particularly in intensifying the competition for talented executives. As the cost of providing signals to these CEOs may have increased, boards could have been potentially discouraged from offering costly compensation plans to them as signals. One implication of compensation disclosure policies, therefore, is that they could moderate executive pay.

This paper also sheds light on the attempts of recent regulations to curb managerial compensation—caps on the bonus-to-salary ratio, for example.[3] I show that if the production function exhibits a certain form of complementarity between matching quality and effort, the principal would increase both bonus and salary in response to the introduction of bonus caps in order to provide signals while maintaining a low bonus-to-salary ratio. The policy, however, also increases firms' signaling costs and could potentially discourage boards from informing executives of their matching quality.

Related Literature. I summarize two main strands of the literature on managerial compensation. One strand (MacLeod, 2003; Levin, 2003; Fuchs, 2015; Zábojník, 2014) studies hidden information, while the other considers moral hazard models (Baker, Gibbons, and Murphy, 1994; Bull, 1987; MacLeod and Malcomson, 1989).

First, this paper is intellectually indebted to the literature on informed principal

[3]EU regulators have decided to institute bonus caps in order to rein in executive compensation. These policies came into effect at the beginning of 2014. Under such a policy, certain bankers can only be paid bonuses equal to their annual salaries or twice as much, if their firm obtains approval from shareholders.

models (Myerson, 1983; Maskin and Tirole, 1992). The general intuition of these models is that when a party that designs a contract has private information, the structure of the contract may reveal some of that information to other parties. Compared to Maskin and Tirole (1992), Myerson (1983) considers a more general setup in which agents also have private information and make private decisions. My paper extends their static setup to a dynamic one and generates new implications regarding compensation structure.

Specifically, my paper contributes to the managerial compensation literature that builds upon informed principal models. MacLeod (2003) generalizes the logic of repeated game models by demonstrating that subjective schemes can be feasible even without infinite interactions, as long as workers can punish a deviation from the implicit contract by imposing some type of socially wasteful cost on the employer. Fuchs (2007) and Zábojník (2014) further develop this model. Fuchs (2015) shows that discretionary salary can be used as a signaling device. The paper focuses on fixed compensation and, thus, leaves aside the moral hazard problem. Zábojník (2014) incorporates the moral hazard problem into Fuchs (2015)'s model. In contrast with my model, the principal in Zábojník (2014)'s model receives a private signal about the agent's expected contribution to the value of the firm in the interim period and, thus, can provide a contractible subjective evaluation to the agent. Neither paper considers a principal whose private information changes dynamically. In addition, I explore the role of incentive pay in providing signals and incentives.

Second, this paper is related to the moral hazard literature. One strand of this literature studies optimal contracts when performance measures are observable to the principal and the agent but are not verifiable. It focuses primarily on how repeated interactions between the principal and the agent help overcome the reneging problem, wherein the principal is tempted to underpay the agent in order to save on labor costs (Baker, Gibbons, and Murphy, 1994; Bull, 1987; MacLeod and Malcomson, 1989). Combining moral hazard and learning, career concern models (Harris and

Holmstrom, 1982; Gibbons and Murphy, 1992) find that increasing explicit incentives can be optimal as the implicit incentives decline over time. As a result, the equilibrium contract provides for wages that do not decline with age. Although these models predict increasing total compensation, they do not explain why every component of CEO compensation is downwardly rigid.

Finally, my paper is related to the strand of the moral hazard literature that studies dynamic contracting problems. According to Lambert (1983), increasing explicit incentives provides insurance to a risk-averse agent in order to reduce the incentive cost. Rogerson (1985) finds that when the principal can dictate the agent's consumption/saving decisions, the optimal consumption pattern tends to be front-loaded. Sannikov (2008) and He (2012) extend previous discrete-time principal-agent models to continuous time and identify the conditions under which the optimal compensation process becomes back-loaded. The literature, however, derives only qualitative predictions regarding total compensation.

1 The Model

The model consists of two periods (period $t = 1$ and 2). There are two players, a principal and an agent.

1.1 Dynamic Environments

In each period, the market condition m_t can be in one of two possible states, $m_t \in \{h, l\}$. h (l) represents a good (bad) market condition.

At the beginning of the first period, the prior probabilities of m_1 being h and l are r and $1 - r$, respectively, and $0 < r < 1$. The market condition might change in the second period. With probability q, a good market condition remains good, $Pr(m_2 = h|m_1 = h) = q$, and $0 < q < 1$. With probability $1 - q$, a good market condition deteriorates, for example, due to increased competition in the product market, $Pr(m_2 = l|m_1 = h) = 1 - q$. Parameters r and q are known to both parties. A bad market condition in the first period remains bad in the second period, $Pr(m_2 = l|m_1 = l) = 1$. This assumption greatly simplifies the analysis by reducing the number of states of the economy. It also provides a robust setting in which to study signaling: If a firm in the bad market does not mimic the firm in the good market, the agent will learn that the firm in the bad market will remain in the bad market in the second period. The firm in the bad market therefore has the strongest incentive to mimic, which gives the firm in the good market the strongest incentive to separate itself from the other firm.

To summarize, the market changes persistently: A good market today predicts a higher likelihood of a good market tomorrow than does a bad market.

1.2 Production Technology

The principal supervises the agent over two periods. The agent's output in each period y_t is verifiable and can take on two possible values, $y_t \in \{0, 1\}$. The probability of achieving output 1 is $p_t = P(\theta_t, e_t)$, a function of matching quality θ_t and the agent's private effort e_t. Depending on the market conditions, θ_t can take two values: θ_l if $m_t = l$, and θ_h if $m_t = h$. Assume that $0 < \theta_l < \theta_h \leq 1$ and $e_t \in [0, 1]$. Output in each period becomes observable only at the end of period 2 (Zábojník, 2014). This assumption simplifies the analysis, as the agent cannot infer matching quality from interim performance but from the contract offered by the principal.

The production technology has the following features: (1) $P(\theta, 0) = 0$ for any θ; (2) $P(\theta, e_t)$ is twice differentiable and increasing in θ and e_t; (3) $\frac{\partial^2 P}{\partial\theta\partial e_t} > 0$; and (4) $P(\theta_h, 1) \leq 1$. Feature (1) means that zero effort leads to zero output. Feature (2) means that the probability of achieving a high output increases with matching quality and effort. Feature (3) means that supermodularity exists between effort and matching quality. In other words, the marginal productivity of the agent's effort increases with the matching quality. The last assumption ensures that the maximum probability of achieving a high output does not exceed 1.

The principal may want to hire the agent even when the matching quality is low. First, the search cost of a high-quality match can be high, and the firm needs a stop-gap agent to work for the firm. Second, in my model, the principal could still make a positive profit by hiring an agent with low matching quality.

1.3 An Informed Principal

As noted in the introduction, the principal is better informed than the agent about matching quality for various reasons. By virtue of monitoring many inputs, a supervisor gains superior information about the worker's talents (Alchian and Demsetz, 1972).

At the beginning of each period, the principal privately receives a perfect signal η_t about the productivity in that period, $\eta_t \in \{l, h\}$. The agent, however, does not observe signals in either period.[4] The principal will decide whether to convey her private information to the agent at the beginning of the first period. Due to the non-observability of the signal to the agent, it is impossible to write a contract contingent on the signal.

1.4 Preferences

The principal and the agent are risk-neutral. For simplicity, I assume that the discount rate for future payoffs is zero. The principal's goal is to maximize the firm's expected profit after deducting the compensation paid to the agent.

The agent's effort cost function is $\psi(e_t)$ for either period. It is twice differentiable in e_t. Assume that $\psi(0) = 0$, and $\psi'(e) > 0$. The agent maximizes the expected compensation after deducting effort disutility. I further assume that $\frac{\partial^2 P}{\partial^2 e} - \psi''(e) < 0$.[5]

[4] Unlike the agent in MacLeod (2003) who receives a private signal, the agent in this model does not receive private signals to abstract from the problem of opinion disagreement between the principal and the agent.

[5] This assumption ensures that the second-order condition of the agent's maximization program is satisfied.

The agent has zero initial wealth and is protected by limited liability. The agent's reservation utility is assumed to be zero for all θ.[6]

1.5 Contract

Here, I characterize the contracting space. Based on the private signal η_1, the principal offers the agent a contract $\mathcal{C}_{t=1}$ at the beginning of the first period. While output is contractible, neither the principal's private information nor the agent's effort is. The contract $\mathcal{C}_{t=1}$ is a subset of $\mathbb{R}^4_+$, $\mathcal{C}_{t=1} \subseteq \mathbb{R}^4_+$. It consists of a set of compensation plans. I call $c^i_{t=2}$, an element of $\mathcal{C}_{t=1}$, a compensation plan. Note that $\mathcal{C}_{t=1}$ may contain more than one compensation plan, i.e., $\mathcal{C}_{t=1} = \{c^i_{t=2}, i = 1, 2, ...n\}$, where $c_{t=2} = \{w(0,0), w(1,0), w(0,1), w(1,1)\}$. The principal pays the agent $w(0,0)$ if $(y_1, y_2) = (0,0)$, $w(1,0)$ if $(y_1, y_2) = (1,0)$, $w(0,1)$ if $(y_1, y_2) = (0,1)$, and $w(1,1)$ if $(y_1, y_2) = (1,1)$. Limited liability constraints imply that all payments are non-negative. The principal commits to the contract $\mathcal{C}_{t=1}$. After receiving the private signal η_2 at the beginning of the second period, the principal, at her sole discretion, chooses a single compensation plan $c^i_{t=2}$ from $\mathcal{C}_{t=1}$.

Because this setting involves a signaling problem, the payment scheme will fully reveal the principal's private information under a separating Perfect Bayesian Equilibrium (PBE). Also, because this PBE setting might have multiple equilibria, I apply Cho and Kreps (1987)'s Intuitive Criterion to refine separating PBEs. For the purpose of this analysis, I focus on separating PBEs, as they are the most interesting ones. I also prove that a pooling equilibrium does not survive the Intuitive Criterion.

1.6 Timing

Figure 1 presents the timeline. At the beginning of the first period, the principal is privately informed of the market condition m_1 and offers a contract $\mathcal{C}_{t=1}$ to the

[6]In an extension, I analyze the case in which the agent's reservation utility is type dependent.

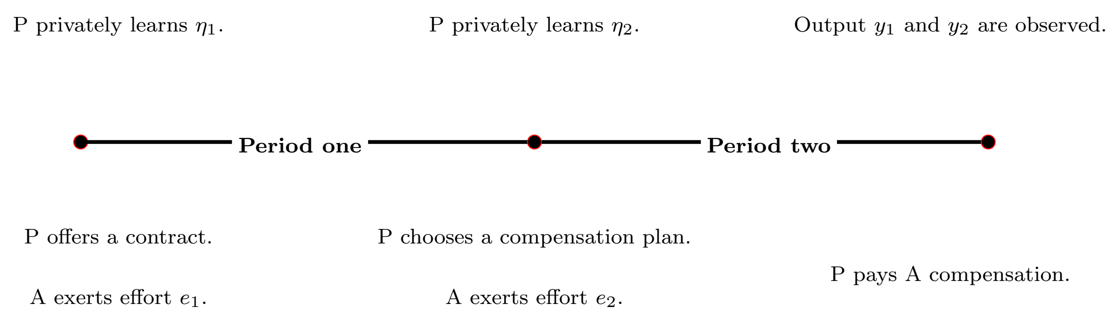

Figure 1: The Timeline

Note: A represents the agent; P represents the principal.

agent. The agent could leave or stay. If he leaves, he obtains a reservation utility of zero. If he accepts the contract, he stays and makes effort decisions. At the beginning of the second period, after observing market condition m_2, the principal chooses a single compensation plan $c^i_{t=2}$ from contract $\mathcal{C}_{t=1}$ and offers it to the agent. Again, the agent could leave or stay. If he leaves, he obtains a reservation utility of zero. If he accepts the compensation plan, he stays and again makes effort decisions. At the end of the second period, the two parties observe the realized values of y_1 and y_2. Finally, compensation is paid.

2 A One-Period Model

Before I characterize the optimal contract in the two-period model, I analyze the one-period model. Figure 2 provides the timeline of the one-period model.

Denote fixed salary as f_1 and performance-based pay as $b_1(y_1)$. While f_1 is paid regardless of performance, $b_1(1)$ is paid when performance $y_1 = 1$ and zero otherwise. The subscript denotes one period. It is easy to show that the contract {w(.)} can be characterized by $\{f_1, b_1(y_1)\}$.[7] Specifically, $f_1 = w(0)$, $b_1(1) = w(1) - w(0)$, and $b_1(0) = 0$.

[7]For the proof, please refer to Lemma 2.

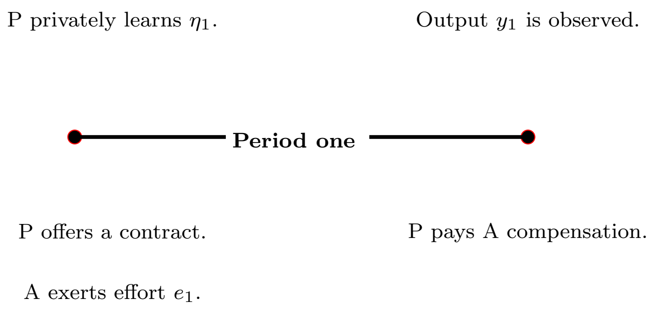

Figure 2: The Timeline

Note: A represents the agent; P represents the principal.

2.1 Symmetric Information

In this section, I analyze the first benchmark case in which the agent receives the the same signal as the principal does in a one-period model. In this benchmark case, both the principal and the agent are informed of the matching quality θ. I use the superscript s to denote compensation $\{f_1^s, b_1^s(y_1)\}$ under symmetric information.

Because the agent is protected by limited liability, the principal cannot punish agent for poor performance. The principal thus chooses to pay the agent the minimum under a low output, that is, $b_1^s(0) = 0$ if $y = 0$. Due to the zero outside option value, the individual participation constraint will be automatically satisfied. Because both parties receive the signal η, there is no need for the principal to provide signal to the agent. As a result, paying f_1^s is not necessary, as it has neither incentive value nor signaling value.

I first analyze the agent's problem. Given the contract, the agent chooses the optimal effort level to maximize his utility:

$$\max_e P(\theta, e)b_1^s - \psi(e)$$

Given the optimal level of effort $e^* = e(\theta, b_1^s)$ as a function of θ and b_1, the principal solves the following maximization program $P0$:

$$\max_{b_1^s} P(\theta_i, e)(1 - b_1^s)$$

$$s.t. \quad e^* = e(\theta, b_1^s) \qquad IC_a$$

$$P(\theta, e)b_1^s - \psi(e) \geq 0 \qquad IR_a$$

Constraint IC_a is the agent's incentive constraint obtained from his own maximization program. Constraint IR_a is the participation constraint of the agent. The limited liability constraint is satisfied if output is low ($y = 0$), as the objective function takes this into account.

The following equation is obtained based on the first-order condition of the principal's maximization program:

$$\frac{\partial P(\theta, e)}{\partial e} \frac{\partial e(\theta, b_1^s)}{\partial b_1^s}(1 - b_1^s) = P(\theta, e^*) \tag{2.1}$$

The left-hand side of Equation 2.1 measures the marginal benefit from an increase in the bonus: It indirectly leads to an increase in the probability of achieving high output through an increase in the agent's effort. The right-hand side of Equation 2.1 represents the marginal cost of an increase in the bonus: It directly increases the expected incentive cost. As argued at the beginning of this section, firms do not pay a salary under symmetric information. In this case, the bonus serves solely the role of incentivizing the agent. The following lemma characterizes the conditions under which the incentive effect becomes weaker or stronger as the production technology varies.

Proposition 1 *Objective incentive compensation under symmetric information:*

- *If* $\frac{\partial^2 lnP(\theta,e)}{\partial\theta\partial e} = 0$, *then* $b_1^{s,h} = b_1^{s,l}$;
- *If* $\frac{\partial^2 lnP(\theta,e)}{\partial\theta\partial e} > 0$, *then* $b_1^{s,h} > b_1^{s,l}$;
- *If* $\frac{\partial^2 lnP(\theta,e)}{\partial\theta\partial e} < 0$, *then* $b_1^{s,h} < b_1^{s,l}$.

Proposition 1 shows that linear supermodularity between matching quality and effort is insufficient to give rise to a bonus that increases in the matching quality. To induce a positive relationship, stronger supermodularity, specifically, positive log-supermodularity is required. Intuitively, if log-supermodularity is positive, the increase in the marginal benefit outweighs the increase in the marginal cost as the bonus increases. I will revisit Proposition 1, as it has important implications for the subsequent analysis.

2.2 Asymmetric Information with an Informative Bonus

This section studies the optimal contract for the second benchmark case: a one-period model with asymmetric information. A separating PBE is defined as follows:

Definition A separating Perfect Bayesian Equilibrium satisfies the following:

1. The principal that hires an agent with matching quality m offers a contract $\{f_1, b_1(y)\}$ that maximizes the firm's profit.

2. The agent's belief regarding the actual matching quality conditional on the contract offered is $\hat{r}(\eta = m|f_1, b_1(y)) = 1$.

3. Given the contract $\{f_1, b_1(y)\}$ and the belief $\hat{r}$, the agent chooses an effort level that maximizes his own utility.

I first analyze the agent's problem. Let $\hat{m}$ be the message that the principal sends to the agent via contract $\{f_1, b_1(y)\}$. The agent chooses an optimal level of effort e to maximize her utility given the contract:

$$\max_{e} P(\hat{m}, e)b_1 + f_1 - \psi(e)$$

From the first-order condition, we obtain the optimal level of effort, which is a function of $\hat{m}$ and b_1, $e^* = e(\hat{m}, b_1)$. That is, the effort level depends on the bonus b_1 and the agent's perceived matching quality or the message $\hat{m}$ sent by the principal.

Given the optimal effort level of the agent, a principal with high matching quality has the following maximization problem $P1$. Here, we solve for the separating equilibrium in which the contract offered by the principal sends a truthful message to the agent: $\hat{r}(m_h|f_1^h, b_1^h) = 1$, and $\hat{r}(m_l|f_1^l, b_1^l) = 1$.

$$\max_{f_1^h, b_1^h} P(\theta_h, e)(1 - b_1^h) - f_1^h$$

$$\begin{aligned}
s.t. \quad & e^* = e(\theta_h, b_1^h) && IC_a \\
& P(\theta_h, e^*)b_1^h - \psi(e^*) + f_1^h \geq 0 && IR_a \\
& P(\theta_h, e(\theta_h, b_1^h))(1 - b_1^h) - f_1^h \geq P(\theta_h, e(\theta_l, b_1^l))(1 - b_1^l) - f_1^l && for \quad IC_h \\
& P(\theta_l, e(\theta_l, b_1^l))(1 - b_1^l) - f_1^l \geq P(\theta_l, e(\theta_h, b_1^h))(1 - b_1^h) - f_1^h && for \quad IC_l
\end{aligned}$$

Constraint IC_a is the agent's incentive constraint obtained from her maximization program. Constraints IC_h and IC_l are truth-telling constraints for principals with high and low matching quality, respectively. These two constraints guarantee that the principal with high matching quality has no incentive to offer the contract offered by the principal with low matching quality, and vice versa.[8] If these constraints are satisfied, the contract that solves the above maximization problem truthfully reveal the matching quality.

To solve for the equilibrium contracts, we first show that if Constraint IC_l is satisfied, then Constraint IC_h is automatically satisfied. Applying Cho and Kreps (1987)'s Intuitive Criterion, the least costly separating equilibrium is the one under which $f_1^l = 0$ and IC_l binds.

Lemma 1 *The principal's problem $P1$ is equivalent to the following maximization problem $P1'$:*

$$\max_{f_1^h, b_1^h} P(\theta_h, e(\theta_h, b_1^h))(1 - b_1^h) - P(\theta_l, e(\theta_h, b_1^h))(1 - b_1^h)$$

[8]Since the production technology exhibits supermodularity, the concavity of the principal's truth-telling constraint is guaranteed.

The optimal bonus under asymmetric information is thus given by the following equation:

$$\left(\frac{\partial P(\theta_h, e(\theta_h, b_1^h))}{\partial e} - \frac{\partial P(\theta_l, e(\theta_h, b_1^h))}{\partial e}\right)\frac{\partial e(\theta_h, b_1^h)}{\partial b_1^h}(1 - b_1^h) = P(\theta_h, e(\theta_h, b_1^h)) - P(\theta_l, e(\theta_h, b_1^h)) \quad (2.2)$$

Similar to Equation 2.1, the left-hand side of Equation 2.2 measures the marginal benefit due to a unit increase in b_1 through an increase in the agent's effort after deducting compensation. The right-hand side of Equation 2.2 represents the associated marginal cost. In contrast with Equation 2.1, it is the sensitivity of output to private information that matters for the characterization of the optimal bonus level, which can be seen from the difference in the two partial derivatives of high and low perceived matching quality in Equation 2.2.

Intuitively, an agent, after receiving a better signal, would work harder, which leads to higher output. When deciding the optimal bonus, a principal with high matching quality would appropriate the profit derived from the agent's improved belief to the agent in order to prevent a principal of lower matching quality from mimicking the higher quality agent. The principal then maximizes profit after deducting the cost associated with signaling.

The following proposition characterizes the condition under which the bonus does not provide signal to the agent:

Proposition 2 ***Information Invariant Condition (IIC)***
If $\partial^2 ln(P(\theta, e) - P(\theta_l, e))/\partial\theta\partial e = 0$, then the bonus is information insensitive, and only the salary provides a signal. $b_1^h = b_1^l = b_1^{s,h} = b_1^{s,l}$, $f_1^h = (P(\theta_l, e(\theta_h, b_1^h) - P(\theta_l, e(\theta_l, b_1^l)))(1 - b_1^l)$, and $f_1^l = 0$.

The IIC condition mutes any effects of information asymmetry on the bonus. The principal fully relies on the salary to provide a signal, while the bonus is paid as if the agent knew her own type (recall Proposition 1). The principal would not want to offer a higher bonus to substitute for the signaling role of the salary, because doing so would imply giving away too much profit.

By offering a salary equal to f_1^h, the principal credibly communicates her private information to the agent, which changes the agent's belief and motivates him to exert more effort. This channel is different from the incentive effect provided by bonus b_1. The salary affects the agent's effort by convincing the agent of his ability to achieve higher output when pay per unit of effort is held constant. The incentive channel affects the agent's effort level by raising b_1 when the agent's belief regarding matching quality is held constant. One direct implication of these two forces is that the salary paid by the principal with high matching quality is increasing in the matching quality of her agent θ_h. This is because the principal with low matching quality has greater potential to gain from mimicking the principal of higher matching quality. The principal with higher matching quality thus has to pay a higher salary to separate herself.

Corollary 1 ***Bonus Providing a Signal***

- *If* $\partial^2 ln(P(\theta,e) - P(\theta_l,e))/\partial\theta\partial e > 0$*, then* $b_1^h > b_1^{s,h}$*, and* $f_1^h = (P(\theta_l, e(\theta_h, b_1^h)) - P(\theta_l, e(\theta_l, b_1^l)))(1 - b_1^h) - P(\theta_l, e(\theta_l, b_1^l))(b_1^h - b_1^l)$;
- *If* $\partial^2 ln(P(\theta,e) - P(\theta_l,e))/\partial\theta\partial e < 0$*, then* $b_1^h < b_1^{s,h}$*, and* $f_1^h = (P(\theta_l, e(\theta_h, b_1^h)) - P(\theta_l, e(\theta_l, b_1^l)))(1 - b_1^h) - P(\theta_l, e(\theta_l, b_1^l))(b_1^h - b_1^l)$.

If the IIC condition is not satisfied, Corollary 1 shows that the bonus can also provide a signal. If $\partial^2 ln(P(\theta,e) - P(\theta_l,e))/\partial\theta\partial e > 0$, better matching quality improves the marginal productivity of effort in terms of the log-likelihood of high output. To prevent a principal with low matching quality from mimicking a principal with high matching quality, the principal with high matching quality pays a bonus that is higher than the level under symmetric information. This is because when the production function exhibits high complementarity between matching quality and effort, signaling through the bonus is cheaper for the principal with high matching quality, as mimicking would involve sharing too much profit with the agent for the principal of low matching quality. This result also features the overprovision of effort compared to the level under symmetric information due to the higher bonus offered. Similar to the case under the IIC condition, the salary can still provide a signal to the agent, as the first term of f_1^h, $(P(\theta_l, e(\theta_h, b_1^h)) - P(\theta_l, e(\theta_l, b_1^l)))(1 - b_1^h)$, is positive. However, the last term of f_1^h, $-P(\theta_l, e(\theta_l, b_1^l))(b_1^h - b_1^l)$, is negative, which implies that the role of salary in providing signal is undermined.

If $\partial^2 ln(P(\theta, e) - P(\theta_l, e))/\partial\theta\partial e < 0$, Corollary 1 shows that the bonus is lower than the level under symmetric information. This result characterizes the condition under which there is underprovision of effort compared to level under symmetric information. In this case, complementarity between matching quality and effort is so insignificant that the firm finds it less costly to use underprovision of effort as a signal. This is because mimicking would induce too little effort for the principal of low matching quality.

3 A Two-period Model

In the two-period model, I choose a specific production technology that satisfies the IIC condition of zero log-supermodularity between matching quality and effort in the one-period model: $P(\theta, e_t) = \theta e$. I further assume that the agent has a quadratic disutility function $\psi(e) = \frac{1}{2}e^2$.

Based on Proposition 1 and Proposition 2, one can easily verify that the optimal bonus offered with such a production function is information insensitive in the one-period model under both symmetric and asymmetric information. This production function greatly simplifies the interpretation of the analysis for a two-period model, because any subsequent changes in bonus that makes it different from the level under a one-period model is not due to a change in the matching quality but rather due to long-term contracting. I will elaborate on this point later.

Long-term contracts are beneficial to the principal in two ways. First, she could signal her private information by using a bonus based on the second-period output measure. In other words, the principal's contracting space that can be used for signaling is expanded. Second, the principal could use cross-pledging based on two period payoffs to alleviate the incentive problem. Because the focus of the paper is on the signaling role of compensation, I first proceed by considering the case in which cross-pledging based on two periods' payoffs is not allowed, from which I obtain the main mechanism. No cross-pledging is perhaps an extreme case, but it corresponds to scenarios in which the firm cannot offer equity, for example, when shareholders are not satisfied with CEO compensation. For completeness, I then characterize the optimal contract if cross-pledging is allowed. Note that the main mechanism is robust to cross-pledging.

Lemma 2 *Contract* $\mathcal{C} = \{w^{hh}_{00}, w^{hh}_{10}, w^{hh}_{01}, w^{hh}_{11}; w^{hl}_{00}, w^{hl}_{10}, w^{hl}_{01}, w^{hl}_{11}\}$ *can be alternatively characterized by* $\mathcal{C} = \{f^{hh}, b^{hh}_1(y_1), b^{hh}_2(y_2), b^{hh}_3(y_1, y_2); f^{hl}, b^{hl}_1(y_1), b^{hl}_2(y_2), b^{hl}_3(y_1, y_2)\}$, *where the superscripts denote the matching quality over two periods.* f *is paid if* $y_1 = y_2 = 0$, $b_1(1) + f$ *is paid if* $y_1 = 1$ *and* $y_2 = 0$, $b_2(1) + f$ *is paid if* $y_1 = 0$ *and* $y_2 = 1$, *and* $b_1(1) + b_2(1) + b_3(1, 1) + f$ *is paid if* $y_1 = y_2 = 1$.

According to Lemma 2, a general contract can be characterized by a specification that consists of fixed pay and variable pay. Fixed pay does not depend on performance. Variable pay is contracted upon different combinations of realized output measures. This specification offers a convenient interpretation of the compensation structure.

3.1 Benchmark Models

First consider a third benchmark case in which information is symmetric in a two-period model. The optimal contract is obvious to derive. Because there is no need to provide signals due to symmetric information, salary is zero. Also, because cross-pledging is not allowed, the bonus for each type of matching quality in each period is the same as the amount provided in the one-period model.

Then consider the fourth benchmark case in which information is asymmetric but committing to a long-term contract is impossible. The following lemma analyzes the contract in this benchmark case; it shows that the optimal contract is stationary in the sense that the bonus does not depend on the underlying matching quality in the earlier period.

Lemma 3 *If committing to a long-term contract is impossible, the optimal contract can be characterized by two one-period contracts.*

- *The first one-period contracts for $m_1 = h$ and $m_1 = l$ at $t = 1$ are:*
  For $m_1 = h$, $\{f_1 = \frac{1}{4}\theta_l(\theta_h - \theta_l), b_1(1) = \frac{1}{2}\}$.
  For $m_1 = l$, $\{f_1 = 0, b_1(1) = \frac{1}{2}\}$.

- *The second one-period contracts for $m_2 = h$ and $m_2 = l$ at $t = 2$ are:*
  For $m_2 = h$, $\{f_2 = \frac{1}{4}\theta_l(\theta_h - \theta_l), b_2(1) = \frac{1}{2}\}$.
  For $m_2 = l$, $\{f_2 = 0, b_2(1) = \frac{1}{2}\}$.

According to Lemma 3, when commitment is impossible, private information in the first period does not affect the equilibrium contract in the second period. When new information arrives in the second period, the principal makes the same offer irrespective of the matching quality in the first period. Specifically, the contract for a low type in the second period who was a low type in the first period is the same as the contract for a low type who was a high type in the first period. Therefore, without commitment, bonus never increases either when the matching quality continues to be good or when it deteriorates, and salary goes down if the matching quality deteriorates.

3.2 Without Cross-pledging

What if commitment is possible in a two-period model with information asymmetry? In addition to sending a positive signal at the beginning of the first period through salary, the principal can also promise more profit sharing through bonuses even if matching quality deteriorates in the second period. The more profit sharing (the higher the bonus) the principal offers, the greater effort the contract could induce due to a greater incentive effect.

In Lemma 4, when cross-pledging is not allowed, the principal cannot use equity compensation that is contracted upon both y_1 and y_2. Bonuses have to be contracted upon y_1 and y_2 separately and are positive in order to offer incentives. The agent's incentive problems in the two periods are tied only through the principal's truth-telling constraint, not through the incentive constraints. To satisfy the limited liability constraints, salaries are non-negative and may be positive to provide signals.

Lemma 4 *When cross-pledging is impossible but commitment is possible,* $b_3^{hh}(1,1) = b_3^{hl}(1,1) = b_3^{ll}(1,1) = 0$. *To induce effort,* $b_1^{hh} = b_1^{hl} > 0$, $b_2^{hh}(1) > 0$, $b_2^{hl}(1) > 0$, $b_1^{ll}(1) > 0$ *and* $b_2^{ll}(1) > 0$. *To satisfy limited liability,* $f^{hl} \geq 0$, $f^{hh} \geq 0$, *and* $f^{ll} = 0$.

Proposition 3 presents the optimal contracts under a separating equilibrium that survives Intuitive Criterion, each of which is unique in a parameter range. it indicates that long-term contracts if commitment is possible depart from the short-term contracts, as commitment allows the principal to reallocate signaling cost over two periods and from salary to bonus.

Proposition 3

- ***Low Separating Profit*** *($\theta_h < 2\theta_l$). The principal of matching quality θ_h commits to a contract in the first period:* $\{b_1^{hh} = \frac{1}{2},\ b_2^{hh} = \frac{1}{2},\ f^{hh} = \frac{1}{4}\theta_l\theta_h - \frac{1}{4}\theta_l^2(2 - \frac{\theta_h}{\theta_l});\ b_1^{hl} = \frac{1}{2},\ b_2^{hl} = \frac{1}{2}(1 + \sqrt{\frac{\theta_h}{\theta_l} - 1}),\ f^{hl} = 0\}$. *The principal offers two one-period contracts to an agent of matching quality θ_l in the first period,* $b_1^{ll} = b_2^{ll} = \frac{1}{2}$.

- ***High Separating Profit*** *($\theta_h \geq 2\theta_l$). The principal of matching quality θ_h commits to a contract in the first period:* $\{b_1^{hh} = \frac{1}{2},\ b_2^{hh} = \frac{1}{2},\ f^{hh} = \frac{1}{2}\theta_l(\theta_h - \theta_l);\ b_1^{hl} = \frac{1}{2},\ b_2^{hl} = 1,\ f^{hl} = \frac{1}{4}\theta_l(\theta_h - 2\theta_l)\}$. *The principal offerd two one-period contracts to an agent of matching quality θ_l in the first period,* $b_1^{ll} = b_2^{ll} = \frac{1}{2}$.

According to Proposition 3, if $\theta_h < 2\theta_l$, then the principal commits to a contract that consists of two compensation plans: one offers a positive salary f^{hh}, the other

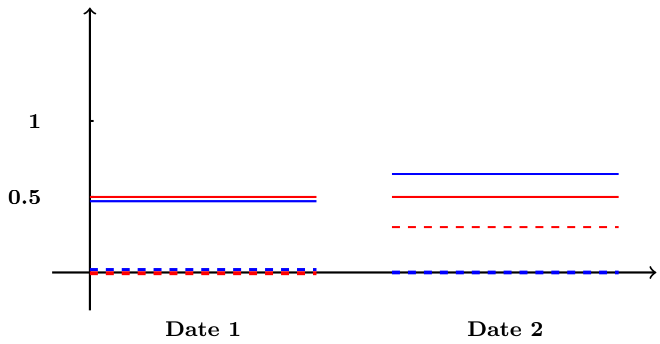

Figure 3: Equilibrium Contracts under Low Separating Profit

Note: Dashed line: salary; Line; bonus; Red: $\theta_h \to \theta_h$; Blue: $\theta_h \to \theta_l$.

offers a bonus b_2^{hl} higher than the other plan but no salary $f^{hl} = 0$.[9] Depending on the signal in the second period, the principal chooses one of the two compensation plans. If the matching quality continues to be good, the principal chooses the contract that offers a high salary, as the salary has signaling value. If the matching quality deteriorates, the principal chooses the contract that offers a higher bonus but no salary. This is because salary has neither incentive nor signaling value when matching quality deteriorates. Thus, a long-term contract with $f^{hl} > 0$ will be subject to renegotiation. Anticipating this, the principal in the first period will substitutes the salary (thus $f^{hl} = 0$) with a higher bonus b_2^{hl} in the second period.

Figure 3 depicts the equilibrium contracts when $\theta_h < 2\theta_l$. Dashed line represents salary, and solid line represents bonus. Red color is for constantly high matching quality, and blue is for deteriorating matching quality. One can see that the contract is downwardly rigid in the bonus and salary under both scenarios.

If $\theta_h \geq 2\theta_l$, according to Proposition 3, the principal commits to a contract that also consists of two compensation plans: one offers a high salary f^{hh}, the other offers a bonus b_2^{hl} that is higher than the other plan but a positive salary that is lower than the other plan. Depending on the signal in the second period, the principal chooses one of the two compensation plans. If the matching quality continues to be good, the

[9] The bonus is one if $\theta_h = 2\theta_l$.

principal chooses the contract that offers a higher salary, as the salary has signaling value. If the matching quality deteriorates, the principal chooses the contract that offers a higher bonus b_2^{hl}.

Like the contract if $\theta_h < 2\theta_l$, the contract if $\theta_h \geq 2\theta_l$ also pays an increasing bonus if the matching quality deteriorates. Unlike the contract if $\theta_h < 2\theta_l$, the contract if $\theta_h \geq 2\theta_l$ pays a positive salary even if the matching quality deteriorates. Because $\theta_h \geq 2\theta_l$, the mimicking incentive for the principal with low matching quality in the first period is greater than when $\theta_h < 2\theta_l$. Thus, the principal with high matching quality in the first period would incur a greater signaling cost to separate: The principal with high matching quality in the first period pays a positive salary even if she receives a bad signal in the second period. Specifically, the contract offers a salary $f^{hl} = \frac{1}{4}\theta_l(\theta_h - 2\theta_l)$ when matching quality deteriorates. The greater the mimicking incentive (i.e., the higher $(\theta_h - 2\theta_l)$), the greater the salary f^{hl}.

If $\theta_h \geq 2\theta_l$, the salary can be paid in the following way: $\frac{1}{8}\theta_l(\theta_h - 2\theta_l)$ in the first period and the same amount in the second period, for a principal with deteriorating matching quality; $\frac{1}{8}\theta_l(\theta_h - 2\theta_l)$ in the first period and $\frac{1}{8}\theta_l(3\theta_h - 2\theta_l)$ in the second period, for a principal with constantly high matching quality. Therefore if matching quality continues to be good, the principal chooses a contract that offers an even higher salary to send a stronger signal. f^{hh} is sufficiently high to prevent the principal with deteriorating matching quality from mimicking. The intuition is that salary is more credible as a signal, as it has zero incentive value and is thus more costly. Figure depicts the equilibrium contracts if $\theta_h \geq 2\theta_l$. One can see that the contract is also downwardly rigid in both salary and bonus.

Discussion. Now recall the contract when commitment is impossible in Lemma 3. The signaling cost in each period is in the form of salary (i.e., $\frac{1}{4}\theta_l(\theta_h - \theta_l)$), and the bonus in each period (i.e., $\frac{1}{2}$) is only to provide incentives. If the matching quality deteriorates, the principal pays a higher rent to the agent in the form of a higher bonus

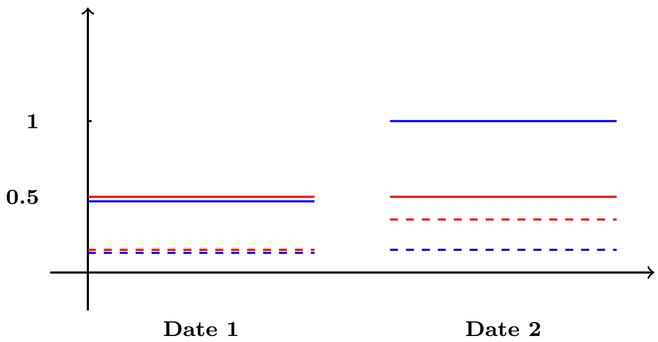

Figure 4: Equilibrium Contracts under High Separating Profit

Note: Dashed line: salary; Line: bonus; Red: $\theta_h \to \theta_h$; Blue: $\theta_h \to \theta_l$.

based on y_2 when commitment is possible than when commitment is impossible. With long-term contracts, the principal is able to reallocate the cost of signaling from salary to the second-period bonus.

To see this more clearly, when commitment is possible, under low separating profit, the principal with constantly high matching quality pays zero salary in the first period and a salary of $\frac{1}{4}\theta_l\theta_h - \frac{1}{4}\theta_l^2(2 - \frac{\theta_h}{\theta_l})$ in the second period, the combined of which is lower than the salary paid by a constantly high matching quality principal when commitment is impossible, i.e., $\frac{1}{2}\theta_l(\theta_h - \theta_l)$. Under low separating profit, the principal with deteriorating matching quality pays zero salary in both periods, which is also lower than the salary paid by a principal with deteriorating matching quality when commitment is impossible, i.e., $\frac{1}{4}\theta_l(\theta_h - \theta_l)$. Bonus based on y_2 paid by the principal with deteriorating matching quality, i.e., $\frac{1}{2}(1 + \sqrt{\frac{\theta_h}{\theta_l} - 1})$, is, however, higher than the bonus paid by the same type of principal when commitment is impossible, i.e., $\frac{1}{2}$. In other words, promising a higher bonus based on second-period performance allows the principal to save signaling cost in the form of salary.

Similarly, when commitment is possible, under high separating profit, the principal with deteriorating matching quality pays a salary of $\frac{1}{4}\theta_l(\theta_h - 2\theta_l)$, which is lower than the salary paid by the same type of principal when commitment is impossible, i.e., $\frac{1}{4}\theta_l(\theta_h - \theta_l)$. Bonus based on y_2 paid by the principal with deteriorating matching quality, i.e., 1, is, however, higher than the bonus paid by the same type of principal when commitment is impossible, i.e., $\frac{1}{2}$. Thus, commitment enables the principal to replace part of salary with bonus as signaling, which explains the downward rigidity in both salary and bonus.

For completeness of the analysis, I prove in the following corollary that pooling equilibrium does not survive Intuitive Criterion. The intuition is that a principal with high matching quality would always benefit from a deviation by offering a certain amount of salary that is equilibrium dominated for the principal of low matching quality.

Corollary 2 *If cross-pledging is not allowed, pooling equilibrium does not survive Intuitive Criterion.*

3.3 With Cross-pledging

The previous section analyzes the optimal contract when cross-pledging is not allowed. The contract features downward rigidity in salary and bonus. In this section, I complete the analysis by considering cross-pledging under which incentives are provided via equity compensation, the value of which depends on past cumulated performance. The principal uses b_3^{hh}, b_3^{hl} and b_3^{ll} to alleviate the incentive problem: By shirking in one period, the agent reduces the probability of full success and, consequently, the reward for the effort exerted in the other period.

With cross-pledging, I first consider long-term equilibrium contract under symmetric information. Because signaling is not needed, a salary is not paid.

Lemma 5 *If information is symmetric, the principal offers the following contracts in equilibrium: Principal with constantly high matching quality hh offers* $\{b_1^{hh} = 0, b_2^{hh} = 0, b_3^{hh} = \frac{1}{\theta_h^2}, f^{hh} = 0\}$*; Principal with deteriorating matching quality hl offers* $\{b_1^{hl} = 0, b_2^{hl} = 0, b_3^{hl} = \frac{1}{\theta_l \theta_h}, f^{hl} = 0\}$*; Principal with constantly low matching quality ll offers:* $\{b_1^{ll} = 0, b_2^{ll} = 0, b_3^{ll} = \frac{1}{\theta_l^2}, f^{ll} = 0\}$.

Lemma 5 shows that when cross-pledging is allowed, the principal uses performance-based pay contracted on two output measures to induce effort. In this way, the principal minimizes the rent the agent extracts due to limited liability. In Proposition 4, I characterize the optimal contracts under information asymmetry.

Proposition 4
- *The principal of high matching quality commits to the following contract in the first period:* $\{b_1^{hh} = 0, b_2^{hh} = 0, b_3^{hh} = \sqrt{\frac{1-(1-q)\theta_l^2}{q\theta_h^4}}, f^{hh} = f^{hl} + \theta_l\theta_h^2 b_3^{hh}(1-\theta_h b_3^{hh}); b_1^{hl} = 0, b_2^{hl} = \frac{3q\theta_l^2 - 2q^2\theta_l^2 - q\theta_h + q^2\theta_h - \theta_l^2 + 1}{2q(1-q)\theta_l^2}, b_3^{hl} = \frac{1}{2\theta_l^2} - \frac{\theta_h b_3^{hh}}{2(1-q)\theta_l^2}, f^{hl} = 1 - \theta_l + (\theta_h - \theta_l)\theta_l^2 b_3^{hl}\}$.
- *The principal of low matching quality offers the following contract in the first period:* $\{b_1^{ll} = 0, b_2^{ll} = 0, b_3^{ll} = \frac{1}{\theta_l^2}, f^{ll} = 0\}$.

In Proposition 4, compared to the optimal contract without cross-pledging, the bonus based on the first-period measure is zero and that based on the first- and second-period measure is positive, which reduces rent extraction by the agent. The principal with high matching quality only uses b_3^{hh} and f^{hh} to induce effort and signal her private information. The equilibrium contract has several interesting features.

First, due to information asymmetry, the principal with deteriorating matching quality offers the agent a larger bonus ($b_2^{hl} > 0$) based on the second-period measure under information asymmetry than under information symmetry ($b_2^{hl} = 0$). This can be seen easily by comparing the contract in Proposition 4 with the contract in Lemma 5. Since the effort in the first period is already sunk and salary has no signaling value as the matching quality deteriorates, the principal would cut b_3^{hl} and salary and increase b_2^{hl} as a signal for high matching quality in the first period. Consequently, the compensation offered by a principal with deteriorating matching quality pays more compensation based on the second performance measure.

Second, $b_3^{hh} > \frac{1}{\theta_h^2}$. The principal with constantly high matching quality offers greater long-term equity compensation under information asymmetry than under information symmetry. As noted above, this is because the agent knows that if the matching quality declines, the principal will offer a bonus based on the second-period output measure. Expecting this, the first-period incentive of an agent with constantly high matching quality would be weakened if the principal did not raise b_3^{hh}, as b_3^{hh} also depends on the first-period performance.

These features imply that the equilibrium contract when cross-pledging is allowed is also downwardly rigid in salary and incentive pay. The principal with high matching quality in the first period can promise to pay at least $\frac{1}{2}f^{hl}$ in each period. If the matching quality improves, she pays more by $f^{hh} - f^{hl}$. The incentive pay offered by the principal with high matching quality in the first period is back loaded and puts more weight on the second-period output measure.

For completeness of the analysis, I also verify that pooling equilibrium does not survive Intuitive Criterion under cross-pledging.

Lemma 6 *If cross-pledging is allowed, pooling equilibrium does not survive Intuitive Criterion.*

4 Transferable Skills and Disclosure Policies

In the previous analysis, the agent's reservation utility does not vary with his matching quality. This represents the case in which an agent's matching quality with the firm does not bring any value outside the firm, or other firms cannot infer the agent's matching quality with them even if it brings value outside that firm. Studies on executive compensation have long been interested in pay-performance sensitivity and its relation with human capital (Murphy and Zábojník, 2007; Dutta, 2008), this section extends the model by assuming type-dependent reservation utility, specifically, the agent's outside option value increases with his matching quality. It corresponds to scenarios in which managerial skills are sufficiently transferable.

This extension also merits consideration in light of mandatory compensation disclosure policy. Current executive compensation disclosure requirements adopted by the US Securities and Exchange Commission (SEC) in 1992 are applicable to most US domestic issuers and to non-US companies that do not qualify as foreign private issuers. The recent Dodd-Frank Wall Street Reform and Consumer Protection Act also contain new disclosure policies that affect the governance of issuers.[10] As the value of an agent's outside option depends on market perception, if firms are required to disclose compensation, then the market will find it easier to assess or infer CEO skills from his compensation. Although this extension does not model labor market competition explicitly, one direct implication of type-contingent reservation utility is that the principal needs to provide higher compensation in order to retain the agent.

The timeline in this extension is the same as that in the one-period baseline model but differs from it in the agent's reservation utility, which is R for the agent of high matching quality and 0 for the agent of low matching quality. As previously, the agent exerts effort $e \in [0, 1]$ with disutility $\psi(e) = \frac{1}{2}e^2$. At the end of date 1, the probability of obtaining $y = 1$ is $p = P(\theta, e) = \theta e$. It can be easily verified that the optimal effort of an agent with matching quality θ_i ($i \in \{l, h\}$) given a contract $\{f_1^i, b_1^i(1)\}$ is $e^{i*} = \theta_i b_1^i$. Thus, the maximization program for a principal who receives a high signal is:

$$\max_{f_1^h, b_1^h} \theta_h e^{h*}(1 - b_1^h) - f_1^h$$

$$s.t. \quad \theta_l e^{l*}(1 - b_1^l) - f_1^l \geq \theta_l e^{h*}(1 - b_1^h) - f_1^h \qquad IC_p$$

$$\theta_h e_h^* b_1^h + f_1^h - \frac{1}{2} e_h^{*2} \geq R \qquad IR_a$$

If $R = 0$, as in the one-period benchmark model, Constraint IR_a is not strictly binding because the agent is protected by limited liability. However, if R is sufficiently large, the surplus the agent extracts due to limited liability may not be large enough to satisfy limited liability. Denote λ as the Lagrangian multiplier of Constraint IR_a.

[10]For instance, Section 953 requires additional disclosure about certain compensation matters, including pay-for-performance and the ratio of the CEO's total compensation to the median total compensation for all other company employees.

Consider the binding case $\lambda > 0$. Under the least costly separating equilibrium, the principal pays $f_1^l = 0$ to an agent of low matching quality, since there is no signaling gain from motivating such an agent. It can be easily verified that $b_1^l = \frac{1}{2}$. A principal who receives a high signal pays the agent only at a level that just makes Constraint IC_p binding. Thus, I substitute b_1^h obtained from Constraint IC_p into the objective function and Constraint IR_a. Proposition 5 compares the bonus of a high matching quality agent with positive reservation utility to the bonus of a high matching quality agent with zero reservation utility.

Proposition 5 ***Bonuses and Managerial Skills***

Assume $b_1^{o,h}$ is the bonus paid to an agent with zero reservation utility and high matching quality, and b_1^h to an agent with positive reservation utility and high matching quality.

- *If $0 \leq R \leq \underline{R}$, Constraint IR_a is not binding ($\lambda = 0$). $b_1^h = b_1^{o,h} = \frac{1}{2}$. Only a separating equilibrium exists.*
- *If $\underline{R} < R \leq \bar{R}$, Constraint IR_a is binding ($\lambda > 0$):*

$$b_1^h = \frac{\Delta\theta + \lambda\theta_l}{2(\Delta\theta + \lambda\theta_l) - \lambda\theta_h}$$

  and $b_1^h > b_1^{o,h} = \frac{1}{2}$. Only a separating equilibrium exists.
- *If $R > \bar{R}$, only a pooling equilibrium exists.*

Proposition 5 indicates that when the agent possesses general skills and her compensation is subject to mandatory disclosure, the agent receives a greater bonus. When the reservation utility for the high type is zero or sufficiently small, the contract could still induce the second-best effort ($b_1^{o,h} = \frac{1}{2}$) under the IIC condition. This is because the rent that the agent extracts due to limited liability is greater than the value of her outside option. When the reservation utility is too high, the principal no longer finds it profitable to provide a signal to the agent. Instead, she chooses to pool with the principal with low matching quality. When the reservation utility is at an intermediate level, the IR_a constraint binds. Having general skills implies higher performance sensitivity ($b_1^h > b_1^{o,h} = \frac{1}{2}$). One might set the bonus at $\frac{1}{2}$ and increase the salary so that Constraint IR_a binds. However, this is not optimal, as salary has no incentive value and is thus more costly to satisfy the participation constraint.

Mandatory disclosure policies thus have two effects. First, when skills are not sufficiently transferable, the principal has to offer higher pay-performance-sensitivity to the agent under mandatory disclosure policies. Second, when skills are sufficiently transferable, the principal chooses not to provide signal. The second effect is consistent with the finding that instituting disclosure policies regarding the value of CEO option grants was followed by a moderation in executive pay in the late 2000s (Shue and Townsend, 2017; Frydman and Jenter, 2010).

5 Bonus Caps and Efficiency Implications

A banker bonus cap was passed by the European Parliament (EP) in April 2013 and took effect in January 2014. In February 2014, the EP and the European Council (the Council), agreed to restrict the bonuses of retail asset managers.[11] In this section, I study its impact on CEO compensation structure from an information asymmetry perspective.

The timeline is the same as in the one-period benchmark model. After observing the contract, the agent exerts effort $e \in [0, 1]$ with disutility $\psi(e) = \frac{1}{2}e^2$. An output y is realized at the end of date 1, and $y \in \{0, 1\}$. In this section, I consider one form of production technology that does not conform to the IIC condition: $P(\theta, e) = \theta_i e(\theta_i + \frac{1}{2}ke)$. This production function has negative log-supermodularity in matching

[11] The Council agreed not to include a bonus cap for managers and advisors of UCITS funds (UCITS funds are similar to US-registered mutual funds). In place of the cap, the Council and EP resolved that at least 50% of the bonus amount must be paid in shares of the fund under management, and at least 40% of the bonus amount must be deferred for three years.

quality and effort. I use such a production function to illustrate potential unintended policy implications, as firms may exhibit heterogeneity in production technology.

	Without a cap	With a cap	Change salary only
Bonus over salary	2.5955	2.5939	2.5939
Salary	0.3415	0.3431	0.3417
Bonus	0.8863	0.89	0.8863
Profit of the firm	0.6294	0.6293	0.6292

Table 1: A Comparison of Efficiency

Note: Parameter values are $k = 1$, $\theta_h = 0.9$, $\theta_l = 0.4$.
The first and second columns show the optimal contracts without and with a cap on the bonus-to-salary ratio, respectively. The third column characterizes the optimal contract under bonus caps by allowing adjustment in salary only.

In Table 1, I present a numerical example using the following parameter values: $k = 1$, $\theta_h = 0.9$, $\theta_l = 0.4$. Further assume that the ratio of bonus to salary cannot exceed 2.5939. The first column is the contract that a principal of high matching quality offers without a cap on the bonus-to-salary ratio. The second and third columns represent the contracts that the principal of high matching quality offers under a cap on the bonus-to-salary ratio. The second column presents the contract that offers the highest possible bonus under the cap. The third column is an alternative contract that adjusts salary only to satisfy the cap while keeping the bonus the same as under no bonus cap.

Comparing the bonuses and salaries in the first and second columns, we find that with a cap on the ratio of bonus to salary, the board increases the bonus and, consequently, the salary, to achieve a lower bonus-to-salary ratio. The board has to pay a greater signaling cost in order to satisfy the limit on bonus-to-salary ratio, and the CEO benefits from the bonus cap. One might argue that the board could consider increasing only the salary. As shown in the third column, such an approach may lead to greater profit destruction for the principal compared to the second approach in the second column, because signaling via salary alone is more costly, as explained in Corollary 1.

My model potentially explains part of the incentives behind compensation adjustments in the industry. Some banks have restructured their CEO compensation by increasing the base salary, resulting in higher estimated total pay. For instance, HSBC's chief executive, Stuart Gulliver, received a salary increase from £1.2 million to £2.9 million thanks to a £32,000 weekly shares of "fixed pay allowance"in 2013. The rational is that the policy may exacerbate the information problem by making truth-telling more costly or even impossible. A principal of high matching quality may find it more profitable not to provide a signal and to pool with the other type of principal.

To conclude, because bonus caps may have heterogeneous impacts on firms with different production technologies, determining the appropriate bonus-to-salary ratio is important. If not, firms would find it difficult to motivate and retain talented executives.

6 Conclusion

This paper characterizes the optimal contract offered by a principal who knows more than the agent about his changing matching quality with the firm. Contracts have two roles: signaling and incentive provision. In the one-period benchmark model, I first present the condition of zero log-supermodularity of the principal's production function in matching quality and effort. Under this condition, the principal solely relies on salary to signal her private information to the agent. Bonuses can also be information sensitive if the condition is not met. Thus, bonuses could play a dual role by providing signals as well as incentives. When the bonus is used to provide a signal, the principal either uses profit sharing (high bonus) or underprovision of effort (low bonus) to signal her private information.

In the two-period model, I choose a specific production function that satisfies the condition. I first analyze a benchmark case in which the principal cannot commit to long-term contracts. Because the agent anticipates that the principal will make a new take-it-or-leave-it offer when new information arrives, he will not agree to an arrangement that promises a high bonus in future as signaling. The equilibrium contract is thus stationary in the sense that the second-period contract does not depend on the first-period private information. If commitment is possible, the principal could promise a higher bonus based on the second-period output as signaling if the matching quality deteriorates. If matching quality continues to be high, the principal wants to provide an even higher salary to provide a signal. The principal pays more bonus to the agent based on the second-period output in exchange for less paid to the agent in the form of salary, as salary has neither signaling nor incentive value when the matching quality deteriorates. Such a contract is non-decreasing in both salary and bonus.

This paper also sheds light on disclosure policies and regulations targeted at curbing managerial compensation. I first consider an extension in which the manager possesses general skills. It suggests that when managerial skills are sufficiently transferable, the principal may choose not to provide a signal due to the excessive signaling cost. I then consider regulations capping the bonus-to-salary ratio. I find that under some production function, the principal may have to increase the bonus to meet the requirement, because an increase in the bonus implies an even greater increase in the salary. Bonus caps thus may exacerbate the information problem by making truth-telling more costly or even impossible.

Several conclusions can be further drawn from this paper. First, while the previous literature focuses mainly on total compensation, this paper emphasizes that compensation structure is an important means for the board to convey information to the CEO. It therefore may carry information content that is predictive of a firm's future performance. Second, non-performance based pay may help facilitate communication in organizations, especially in situations where using performance-based pay as signaling is too costly.

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Appendix Proof

Proposition 1. Objective incentive compensation under symmetric information:

- If $\frac{\partial^2 lnP(\theta,e)}{\partial\theta\partial e} = 0$, then $b_1^{s,h} = b_1^{s,l}$;
- If $\frac{\partial^2 lnP(\theta,e)}{\partial\theta\partial e} > 0$, then $b_1^{s,h} > b_1^{s,l}$;
- If $\frac{\partial^2 lnP(\theta,e)}{\partial\theta\partial e} < 0$, then $b_1^{s,h} < b_1^{s,l}$.

Proof The principal sets an optimal level of performance-based pay to maximize the expected profit, which is a function of the matching quality (θ), the performance-based pay ($b_1(\theta)$), and the optimal effort that the agent chooses given the matching quality and the performance-based pay ($e^*(\theta, b_1(\theta))$).

$$\max_{b_1} \quad u = U(\theta, e^*(\theta, b_1(\theta)), b_1(\theta))$$

The first order derivative is thus:

$$F.O.C. \quad \frac{\partial u}{\partial b_1} = \frac{\partial U}{\partial e}\frac{\partial e}{\partial b_1} + \frac{\partial U}{\partial b_1} = 0 \tag{7.1}$$

Take first order derivative of Equation 7.1 w.r.t. θ:

$$\frac{\partial^2 U}{\partial b_1 \partial\theta} + \frac{\partial e}{\partial b_1}\frac{\partial^2 U}{\partial e\partial\theta} + (\frac{\partial^2 U}{\partial b_1\partial e}\frac{\partial e}{\partial b_1} + \frac{\partial^2 U}{\partial^2 b_1} + \frac{\partial^2 U}{\partial^2 e}(\frac{\partial e}{\partial b_1})^2 + \frac{\partial e}{\partial b_1}\frac{\partial^2 U}{\partial e\partial b_1} + \frac{\partial U}{\partial e}\frac{\partial^2 e}{\partial^2 b_1}) \times \frac{db_1}{d\theta} = 0$$

Rearrange the equation, I obtain:

$$\frac{db}{d\theta} =(\frac{\partial^2 U}{\partial b_1 \partial\theta} + \frac{\partial e}{\partial b_1}\frac{\partial^2 U}{\partial e\partial\theta})/(\frac{\partial^2 U}{\partial b_1\partial e}\frac{\partial e}{\partial b_1} + \frac{\partial^2 U}{\partial^2 b_1} + \frac{\partial^2 U}{\partial^2 e}(\frac{\partial e}{\partial b_1})^2 + \frac{\partial e}{\partial b_1}\frac{\partial^2 U}{\partial e\partial b_1} + \frac{\partial U}{\partial e}\frac{\partial^2 e}{\partial^2 b_1}) \tag{7.2}$$

Assume the second order condition of the principal's problem is satisfied, thus

$$\frac{\partial^2 U}{\partial b_1\partial e}\frac{\partial e}{\partial b_1} + \frac{\partial^2 U}{\partial^2 b_1} + \frac{\partial^2 U}{\partial^2 e}(\frac{\partial e}{\partial b_1})^2 + \frac{\partial e}{\partial b_1}\frac{\partial^2 U}{\partial e\partial b_1} + \frac{\partial U}{\partial e}\frac{\partial^2 e}{\partial^2 b_1} < 0$$

As a result,

$$\frac{db}{d\theta} = 0 \Leftrightarrow \frac{\partial^2 U}{\partial b_1 \partial \theta} + \frac{\partial e}{\partial b_1}\frac{\partial^2 U}{\partial e \partial \theta} = 0$$

From Equation 7.1, I obtain:

$$\frac{\partial e}{\partial b_1} = -\frac{\partial U}{\partial b_1}/\frac{\partial U}{\partial e}$$

Also because $U(\theta, e^*(\theta, b_1(\theta)), b_1(\theta)) = P(\theta, e)(1 - b_1(\theta))$, I obtain:

$$\frac{\partial U}{\partial b_1} = -P(\theta, e)$$
$$\frac{\partial U}{\partial e} = (1-b)\frac{\partial P}{\partial e}$$

I therefore obtain:

$$P\frac{\partial^2 P(\theta, e)}{\partial e \partial \theta} - \frac{\partial P(\theta, e)}{\partial e}\frac{\partial P(\theta, e)}{\partial \theta} = 0$$
$$\Leftrightarrow \quad \frac{\partial^2 lnP(\theta, e)}{\partial e \partial \theta} = 0$$

One example of solutions to the above PDE is $P(\theta, e) = h(\theta)f(e)$.

It easily follows that if $\frac{\partial^2 lnP(\theta,e)}{\partial e \partial \theta} > 0$, $b_1^{s,h} > b_1^{s,l}$. And vice versa. Q.E.D.

Lemma 1. The principal's problem $P1$ is equivalent to the following maximization problem $P1'$:

$$\max_{b_1^h} P(\theta_h, e(\theta_h, b_1^h))(1 - b_1^h) - P(\theta_l, e(\theta_h, b_1^h))(1 - b_1^h)$$

Proof Substituting Constraint IC_a and Constraint IC_l into the objective function, problem $P1$ with two constraints then is simplified to problem $P1'$. Q.E.D.

Proposition 2. If $\partial^2 ln(P(\theta, e) - P(\theta_l, e))/\partial\theta\partial e = 0$, then the bonus is information insensitive, and only the salary provides a signal. $b_1^h = b_1^l = b_1^{s,h} = b_1^{s,l}$, $f_1^h = (P(\theta_l, e(\theta_h, b_1^h) - P(\theta_l, e(\theta_l, b_1^l)))(1 - b_1^l)$, and $f_1^l = 0$.

Proof Following Lemma 1, set the principal's maximization objective as

$$\max_{b_1^h} \quad u = U(\theta_h, e^*(\theta_h, b_1^h), b_1^h) = P(\theta_h, e(\theta_h, b_1^h))(1 - b_1^h) - P(\theta_l, e(\theta_h, b_1^h))(1 - b_1^h)$$

Similar to the proof in Proposition 1, I obtain the derivative of bonus with respect to θ_h:

$$\frac{db_1^h}{d\theta_h} = 0 \Leftrightarrow \frac{\partial^2 U}{\partial b_1^h \partial \theta_h} + \frac{\partial e}{\partial b_1^h}\frac{\partial^2 U}{\partial e \partial \theta_h} = 0$$

Because from Equation 7.1, I obtain:

$$\frac{\partial e}{\partial b_1^h} = -\frac{\partial U}{\partial b_1^h} / \frac{\partial U}{\partial e}$$

Also because $U(\theta, e^*(\theta, b_1(\theta)), b_1(\theta)) = P(\theta_h, e(\theta_h, b_1^h))(1 - b_1^h) - P(\theta_l, e(\theta_h, b_1^h))(1 - b_1^h)$, I obtain:

$$\begin{aligned}\frac{\partial U}{\partial b_1^h} &= -P(\theta_h, e) + P(\theta_l, e) \\ \frac{\partial U}{\partial e} &= (1 - b_1^h)\frac{\partial(P(\theta_h, e) - P(\theta_l, e))}{\partial e}\end{aligned}$$

I therefore obtain:

$$\begin{aligned}(P(\theta_h, e) - P(\theta_l, e))\frac{\partial^2(P(\theta_h, e) - P(\theta_l, e))}{\partial e \partial \theta_h} &= \frac{\partial(P(\theta_h, e) - P(\theta_l, e))}{\partial e}\frac{\partial(P(\theta_h, e) - P(\theta_l, e))}{\partial \theta_h} \\ \Leftrightarrow \quad & \frac{\partial^2 ln(P(\theta_h, e) - P(\theta_l, e))}{\partial e \partial \theta_h} = 0\end{aligned}$$

$P(\theta, e) = h(\theta)f(e)$ again is an example of solutions to the above PDE. Q.E.D.

Corollary 1.

- If $\frac{\partial^2 ln(P(\theta,e) - P(\theta_l,e))}{\partial\theta\partial e} > 0$, then $b_1^h > b_1^{s,h}$, and $f_1^h = (P(\theta_l, e(\theta_h, b_1^h)) - P(\theta_l, e(\theta_l, b_1^l)))(1 - b_1^h) - P(\theta_l, e(\theta_l, b_1^l))(b_1^h - b_1^l)$;
- If $\frac{\partial^2 ln(P(\theta,e) - P(\theta_l,e))}{\partial\theta\partial e} < 0$, then $b_1^h < b_1^{s,h}$, and $f_1^h = (P(\theta_l, e(\theta_h, b_1^h)) - P(\theta_l, e(\theta_l, b_1^l)))(1 - b_1^h) - P(\theta_l, e(\theta_l, b_1^l))(b_1^h - b_1^l)$.

Proof Following the proof in Proposition 2, it can be easily shown that if $\frac{\partial^2 ln(P(\theta,e) - P(\theta_l,e))}{\partial\theta\partial e} >$

0, then $\frac{db_1^h}{d\theta_h} > 0$, thus $b_1^h > b_1^l = b_1^{s,l} = b_1^{s,h}$. Similarly, if $\frac{\partial^2 ln(P(\theta,e)-P(\theta_l,e))}{\partial\theta\partial e} < 0$, then $\frac{db_1^h}{d\theta_h} < 0$, thus $b_1^h < b_1^l = b_1^{s,l} = b_1^{s,h}$.

I then obtain the amount of salary from Constraint IC_l. Q.E.D.

Lemma 2. Contract $\mathcal{C} = \{w_{00}^{hh}, w_{10}^{hh}, w_{01}^{hh}, w_{11}^{hh}; w_{00}^{hl}, w_{10}^{hl}, w_{01}^{hl}, w_{11}^{hl}\}$ can be alternatively characterized by $\mathcal{C} = \{f^{hh}, b_1^{hh}(y_1), b_2^{hh}(y_2), b_3^{hh}(y_1, y_2); f^{hl}, b_1^{hl}(y_1), b_2^{hl}(y_2), b_3^{hl}(y_1, y_2)\}$ where superscripts denote the matching quality over two periods. f is the fixed compensation regardless of the performance. $b_1(1) + f$ is paid if $y_1 = 1$ and $y_2 = 0$, $b_2(1) + f$ is paid if $y_1 = 0$ and $y_2 = 1$, $b_1(1) + b_2(1) + b_3(1,1) + f$ is paid if $y_1 = y_2 = 1$, and f is paid if $y_1 = y_2 = 0$.

Proof At the second period, the agent of matching quality θ_h at date 1 and θ_h at date 2 maximizes effort e_2^{hh} given the contract and first period effort e_1^h.

$$\max_{e_2^{hh}} \; (1-\theta_h e_1^h)(1-\theta_h e_2^{hh})w_{00}^{hh} + \theta_h e_1^h(1-\theta_h e_2^{hh})w_{10}^{hh} + \theta_h e_2^{hh}(1-\theta_h e_1^h)w_{01}^{hh} + \theta_h e_2^{hh}\theta_h e_1^h w_{11}^{hh} - \frac{1}{2}e_2^{hh^2}$$

$$\Longleftrightarrow$$

$$\max_{e_2^{hh}} \; w_{00}^{hh} + \theta_h e_1^h(w_{10}^{hh} - w_{00}^{hh}) + \theta_h e_2^{hh}(w_{01}^{hh} - w_{00}^{hh}) + \theta_h e_2^{hh}\theta_h e_1(w_{11}^{hh} - w_{10}^{hh} - (w_{01}^{hh} - w_{00}^{hh})) - \frac{1}{2}e_2^{hh^2}$$

Set $w_{00}^{hh} = f^h$, $w_{10}^{hh} - w_{00}^{hh} = b_1^{hh}$, $w_{01}^{hh} - w_{00}^{hh} = b_2^{hh}$, and $w_{11}^{hh} - w_{00}^{hh} - (w_{01}^{hh} - w_{00}^{hh}) - (w_{10}^{hh} - w_{00}^{hh}) = b_3^{hh}$. It is easy to see that effort e_2^{hh} is only affected by b_2^{hh} and b_3^{hh}. I obtain the follwing:

$$e_2^{hh} = \theta_h(b_2^{hh} + \theta_h e_1^h b_3^{hh})$$

Likewise, at the second period, the agent of matching quality θ_h at date 1 and θ_l at date 2 maximizes effort e_2^{hl} given the contract and first period effort e_1^h. Set $w_{00}^{hl} = f^{hl}$, $w_{10}^{hl} - w_{00}^{hl} = b_1^{hl}$, $w_{01}^{hl} - w_{00}^{hl} = b_2^{hl}$, and $w_{11}^{hl} - w_{00}^{hl} - (w_{01}^{hl} - w_{00}^{hl}) - (w_{10}^{hl} - w_{00}^{hl}) = b_3^{hl}$. Similar to the above maximization program, the agent makes an effort as follows:

$$e_2^{hl} = \theta_l(b_2^{hl} + \theta_h e_1^h b_3^{hl})$$

When the agent makes effort e_1^h in the first period, neither the principal nor the agent knows the private information in period 2, thus $b_1^{hl} = b_1^{hh}$. The agent with high matching quality thus maximizes the first-period compensation plus the second-period expected compensation.

$$\max_{e_1^h} \quad w_{00}^{hh} + \theta_h e_1^h (w_{10}^{hh} - w_{00}^{hh}) - \frac{1}{2} {e_1^h}^2$$
$$+ q(\theta_h e_2^{hh} (w_{01}^{hh} - w_{00}^{hh}) + \theta_h e_2^{hh} \theta_h e_1^h (w_{11}^{hh} - w_{10}^{hh} - (w_{01}^{hh} - w_{00}^{hh})) - \frac{1}{2} {e_2^{hh}}^2)$$
$$(1-q)(\theta_l e_2^{hl} (w_{01}^{hl} - w_{00}^{hl}) + \theta_l e_2^{hl} \theta_h e_1^h (w_{11}^{hl} - w_{10}^{hl} - (w_{01}^{hl} - w_{00}^{hl})) - \frac{1}{2} {e_2^{hl}}^2)$$

Define $\beta^{hh} = (b_2^{hl} + \theta_h e_1^h b_3^{hh})$ and $\beta^{hl} = (b_2^{hl} + \theta_h e_1^h b_3^{hl})$.

$$e_1^h = \theta_h b_1^h + q\theta_h^3 b_3^{hl} \beta^{hh} + (1-q)\theta_h \theta_l^2 b_3^{hl} \beta^{hl}$$

Now let's turn to the principal's maximization program. Given the agent's effort, the principal maximizes her profit w.r.t. the eight parameters in $\mathcal{C}$.

$$\max_{w\{..\}} \quad q\{-(1-\theta_h e_1^h)(1-\theta_h e_2^{hh}) w_{00}^{hh} + \theta_h e_1^h (1-\theta_h e_2^{hh})(1-w_{10}^{hh})$$
$$+ \theta_h e_2^{hh} (1-\theta_h e_1^h)(1-w_{01}^{hh}) + \theta_h e_2^{hh} \theta_h e_1^h (2-w_{11}^{hh})\}$$
$$+ (1-q)\{-(1-\theta_h e_1^h)(1-\theta_l e_2^{hl}) w_{00}^{hl} + \theta_h e_1^h (1-\theta_l e_2^{hl})(1-w_{10}^{hl})$$
$$+ \theta_l e_2^{hl} (1-\theta_h e_1^h)(1-w_{01}^{hl}) + \theta_l e_2^{hl} \theta_h e_1^h (2-w_{11}^{hl})\}$$

$$\iff \max_{w\{..\}} \quad q\{\theta_h e_1^h (1-(w_{10}^{hh} - w_{00}^{hh})) + \theta_h e_2^{hh} (1-(w_{01}^{hh} - w_{00}^{hh}))$$
$$- \theta_h e_1^h \theta_h e_2^{hh} (w_{11}^{hh} - w_{00}^{hh} - (w_{01}^{hh} - w_{00}^{hh}) - (w_{10}^{hh} - w_{00}^{hh})) - w_{00}^{hh}\}$$
$$+ (1-q)\{\theta_h e_1^h (1-(w_{10}^{hl} - w_{00}^{hl})) + \theta_l e_2^{hl} (1-(w_{01}^{hl} - w_{00}^{hl}))$$
$$- \theta_h e_1 \theta_l e_2^{hl} (w_{11}^{hl} - w_{00}^{hl} - (w_{01}^{hl} - w_{00}^{hl}) - (w_{10}^{hl} - w_{00}^{hl})) - w_{00}^{hl}\}$$

Recall we have set the following variables previously $w_{00}^{hh} = f^h$, $w_{10}^{hh} - w_{00}^{hh} = b_1^{hh}$, $w_{01}^{hh} - w_{00}^{hh} = b_2^{hh}$, and $w_{11}^{hh} - w_{00}^{hh} - (w_{01}^{hh} - w_{00}^{hh}) - (w_{10}^{hh} - w_{00}^{hh}) = b_3^{hh}$; $w_{00}^{hl} = f^{hl}$, $w_{10}^{hl} - w_{00}^{hl} = b_1^{hl}$, $w_{01}^{hl} - w_{00}^{hl} = b_2^{hl}$, and $w_{11}^{hl} - w_{00}^{hl} - (w_{01}^{hl} - w_{00}^{hl}) - (w_{10}^{hl} - w_{00}^{hl}) = b_3^{hl}$ It is obvious to see that the above system can be transferred to the following one under the specification of $\{b\{.\}, f\{.\}\}$:

$$\max_{\{b\{.\},f\{.\}\}} \quad \theta_h e_1^h(1-b_1^h) + q\{\theta_h e_2^{hh}(1-b_2^{hh}) - \theta_h e_1^h \theta_h e_2^{hh} b_3^{hh} - f^h\}$$
$$+ (1-q)\{\theta_l e_2^{hl}(1-b_2^{hl})) - \theta_h e_1^h \theta_l e_2^{hl} b_3^{hl} - f^l\}$$

Q.E.D.

Lemma 3 If committing to a long term contract is impossible, the optimal contract can be characterized by two one-period contracts.

- The first one-period contracts for $m_1 = h$ and $m_1 = l$ at $t = 1$ are:
  For $m_1 = h$, $\{f_1 = \frac{1}{4}\theta_l(\theta_h - \theta_l), b_1(1) = \frac{1}{2}\}$.
  For $m_1 = l$, $\{f_1 = 0, b_1(1) = \frac{1}{2}\}$.
- The second one-period contracts for $m_2 = h$ and $m_2 = l$ at $t = 2$ are:
  For $m_2 = h$, $\{f_2 = \frac{1}{4}\theta_l(\theta_h - \theta_l), b_2(1) = \frac{1}{2}\}$.
  For $m_2 = l$, $\{f_2 = 0, b_2(1) = \frac{1}{2}\}$.

Proof The derivation is straightforward given the proof in Proposition 2. One only needs to substitute the production function $P(\theta, e_t) = \theta e$ and the disutility function $\psi(e) = \frac{1}{2}e^2$ into the principal and the agent's maximization programs and solve for the optimal contracts. Q.E.D.

Lemma 4 When cross-pledging is impossible but commitment is possible, $b_3^{hh}(1,1) = b_3^{hl}(1,1) = b_3^{ll}(1,1) = 0$. To induce effort, $b_1^{hh} = b_1^{hl} > 0$, $b_2^{hh}(1) > 0$, $b_2^{hl}(1) > 0$, $b_1^{ll}(1) > 0$ and $b_2^{ll}(1) > 0$. To satisfy limited liability, $f^{hl} \geq 0$ and $f^{hh} \geq 0$.

Proof 1. Assume the compensation paid to the agent of low matching quality at date 0 is $\{b_1^{ll}, b_2^{ll}, b_3^{ll}\}$. $b_1^{ll} > 0$, if $y_1 = 1$. $b_2^{ll} > 0$, if $y_2 = 1$. $b_3^{ll} > 0$, if $y_1 = y_2 = 1$. The principal does not need to pay salary to this agent in the separating equilibrium. I first prove that if b_3^{ll} is set to zero for the agent of type θ_l at date 0, $b_1^{ll} > 0$, $b_2^{ll} > 0$. To simplify notation, $f^{hl} = f^l$ and $f^{hh} = f^h$.

Composite bonuses β^{hh} and β^{hl} defined in the proof Lemma 2 are important auxiliary variables. Here define $\beta^{ll} = (b_2^{ll} + \theta_l e_1^l b_3^{ll})$, so $b_2^{ll} = \beta^{ll} - \theta_l e_1^{ll} b_3^{ll}$. Following the proof in

Lemma 2, it is easy to prove that $e_2^{ll} = \beta^{ll}\theta_l$ and $e_1^{ll} = \theta_l b_1^{ll} + \theta_l^3 \beta^{ll} b_3^{ll}$. The principal's maximization program is thus:

$$\max_{\{b_1^{ll}, b_2^{ll}, b_3^{ll}\}} \quad \theta_l e_1^{ll}(1-b_1^{ll}) + \theta_l e_2^{ll}(1-b_2^{ll}) - \theta_l e_1^{ll} \theta_l e_2^{ll} b_3^{ll}$$
$$\Leftrightarrow \max_{\{b_1^{ll}, \beta^{ll}, b_3^{ll}\}} \quad \theta_l(\theta_l b_1^{ll} + \theta_l^3 \beta^{ll} b_3^{ll})(1-b_1^{ll}) + \theta_l^2 \beta^{ll}(1-(\beta^{ll} - \theta_l e_1^{ll} b_3^{ll})) - \theta_l^3 \beta^{ll} e_1^{ll} b_3^{ll}$$
$$\Leftrightarrow \max_{\{b_1^{ll}, \beta^{ll}, b_3^{ll}\}} \quad \theta_l^2 b_1^{ll}(1-b_1^{ll}) + \theta_l^2 \beta^{ll}(1-\beta^{ll}) + \theta_l^4 \beta^{ll}(1-b_1^{ll}) b_3^{ll}$$

b_3^{ll} only enters into the maximization program through term $\theta_l^4 \beta^{ll}(1-b_1^{ll})b_3^{ll}$. It can be easily verify that $b_1^{ll} = b_2^{ll} = \frac{1}{2}$.

2. The second step is to prove that $b_1^{hh} = b_1^{hl} > 0$, $b_2^{hh} > 0$, and $f^l \geq 0$ and $f^h \geq 0$. Because in the first period, neither the agent nor the principal knows the second-period matching quality, thus $b_1^{hh} = b_1^{hl} = b_1^h$, and $e_1^{hh} = e_1^{hl} = e_1^h$. A principal who hires an agent of matching quality θ_h at date 0 maximizes the profit subject to two truth-telling constraints. The principal solves the following maximization problem P^h:

$$\max_{\{b\{.\}, f\{.\}\}} \quad \theta_h e_1^h(1-b_1^h) + q\{\theta_h e_2^{hh}(1-b_2^{hh}) - \theta_h e_1^h \theta_h e_2^{hh} b_3^{hh} - f^h\}$$
$$+ (1-q)\{\theta_l e_2^{hl}(1-b_2^{hl})) - \theta_h e_1^h \theta_l e_2^{hl} b_3^{hh} - f^l\}$$

$$s.t. \quad \theta_l e_1^h(1-b_1^h) + \theta_l e_2^{hl}(1-b_2^{hl}) - \theta_l e_2^{hl} \theta_l e_1^h b_3^{hl} - f^l \leq \frac{1}{4}\theta_l^2 + \frac{1}{4}\theta_l^2 \tag{7.3}$$
$$\theta_l e_2^{hh}(1-b_2^{hh}) - \theta_l e_2^{hh} \theta_h e_1^h b_3^{hh} - f^h \leq \theta_l e_2^{hl}(1-b_2^{hl}) - \theta_l e_2^{hl} \theta_h e_1^h b_3^{hl} - f^l \tag{7.4}$$

In addition, the principal does not want to renegotiate the contract at the beginning of the second period when new private information arrives. I'll come back to this point later. Substituting b_2^{hh} and b_2^{hl} with β^{hh} and β^{hl} (see Lemma 2), the above two constraints are equivalent to:

$$f^l = \theta_l e_1^h(1-b_1^h) + \theta_l^2 \beta^{hl}(1-\beta^{hl}) + \theta_l^2(\theta_h - \theta_l) e_1^h \beta^{hl} b_3^{hl} - \frac{1}{2}\theta_l^2$$
$$f^h = \theta_l \theta_h \beta^{hh}(1-\beta^{hh}) - \theta_l^2 \beta^{hl}(1-\beta^{hl}) + f^l$$

In order to satisfy the limited liability, $f^l \geq 0$ and $f^h \geq 0$. The principal's maximization program is equivalent to the following program P'^h:

$$\max_{\{b\{.\},\beta\{.\}\}} \quad (\theta_h - \theta_l)\{e_1^h(1-b_1^h) - \theta_l^2 e_1^h \beta_l b_3^{hl}\} + q(\theta_h - \theta_l)\theta_h \beta^{hh}(1-\beta^{hh}) + \theta_l^2$$

From Lemma 2, we know that:

$$e_1^h = \theta_h b_1^h + q\theta_h^3 b_3^{hh}\beta^{hh} + (1-q)\theta_h \theta_l^2 b_3^{hl}\beta^{hl}$$

It's easy to see that b_3^{hh} only enters into the maximization program through term $(\theta_h - \theta_l)(1-b_1^h)q\theta_h^3\beta^{hh}b_3^{hh}$. Without cross-pledging, it is zero. Thus $b_1^{hh} = b_1^{hl} = b_1^h > 0$ to induce first period effort. Similarly, one can find that $b_2^{hh} > 0$ to induce second period effort.

3. The last step is to prove that $b_2^{hl} > 0$. The principal deteriorating matching quality in the second period may want to renegotiate the contract. Assume if renegotiation happens, the renegotiated contract specification is given by $\{b_2'^{hl}, b_3'^{hl}, f'^l\}$. Define $\beta'^{hl} = (b_2'^{hl} + \theta_h e_1^h b_3'^{hl})$, thus effort $e_2'^{hl} = \theta_l \beta'^{hl}$. A renegotiation-proof contract must satisfy the following maximization program P^{hl}:

$$\begin{aligned} \{b_2^{hl}, b_3^{hl}, f^l\} \quad &\in arg \max_{\{b_2'^{hl}, b_3'^{hl}, f'^l\}} \theta_l e_2'^{hl}(1-b_2'^{hl}) - \theta_h e_1^h \theta_l e_2'^{hl} b_3'^{hl} - f'^l \\ s.t. \quad &\theta_l e_2'^{hl}(b_2'^{hl} + \theta_h e_1^h b_3'^{hl}) + f'^l - \frac{1}{2}e_2'^{hl2} \geq \theta_l e_2^{hl}(b_2^{hl} + \theta_h e_1^h b_3^{hl}) + f^l - \frac{1}{2}e_2^{hl2} \end{aligned}$$

Assume $u = \theta_l e_{2l}(b_{2l} + \theta_h e_1 b_{3l}) + f_l - \frac{1}{2}e_{2l}^2$, the above program is equivalent to the following program P'^{hl}:

$$\begin{aligned} \{\beta^{hl}, b_3^{hl}, f^l\} \quad &\in arg \max_{\{\beta'^{hl}, b_3'^{hl}, f'^l\}} \theta_l^2 \beta'^{hl}(1-\beta'^{hl}) - f'^l \\ s.t. \quad &\frac{1}{2}\theta_l^2 \beta'^{hl2} - f'^l \geq u \end{aligned}$$

b_3^{hl} thus does not enter the renegotiation-proof contract. To induce second period effort, b_2^{hl} must be greater than zero. Q.E.D.

Proposition 3.

- **Low Separating Profit** ($\theta_h < 2\theta_l$). The principal of matching quality θ_h commits to a contract at date 0: $\{b_1^{hh} = \frac{1}{2},\ b_2^{hh} = \frac{1}{2},\ f^{hh} = \frac{1}{4}\theta_l\theta_h - \frac{1}{4}\theta_l^2(2 - \frac{\theta_h}{\theta_l});\ b_1^{hl} = \frac{1}{2},\ b_2^{hl} = \frac{1}{2}(1 + \sqrt{\frac{\theta_h}{\theta_l} - 1}),\ f^{hl} = 0\}$. The principal will offer two one-period contracts to the agent of matching quality θ_l at date 0, $b_1^{ll} = b_2^{ll} = \frac{1}{2}$.

- **High Separating Profit** ($\theta_h \geq 2\theta_l$). The principal of matching quality θ_h commits to a contract at date 0: $\{b_1^{hh} = \frac{1}{2},\ b_2^{hh} = \frac{1}{2},\ f^{hh} = \frac{1}{2}\theta_l(\theta_h - \theta_l);\ b_1^{hl} = \frac{1}{2},\ b_2^{hl} = 1,\ f^{hl} = \frac{1}{4}\theta_l(\theta_h - 2\theta_l)\}$. The principal will offer two one-period contracts to the agent of matching quality θ_l at date 0, $b_1^{ll} = b_2^{ll} = \frac{1}{2}$.

Proof To simplify notation, $f^{hl} = f^l$ and $f^{hh} = f^h$. The principal of high matching quality in the first period offers an optimal contract that is renegotiation-proof. A renegotiation-proof contract must satisfy the maximization program P^{hh} for a principal of matching quality hh listed in step 1 and program P^{hl} for a principal of matching quality hl listed in step 2:

1. Program P^{hh}

$$\{\beta^{hh}, b_3^{hh}, f^h\} \in arg \max_{\{b_2'^{hh}, b_3'^{hh}, f'^h\}} \theta_h e_2'^{hh}(1 - b_2'^{hh}) - \theta_h e_1^h \theta_h e_2'^{hh} b_3'^{hh} - f'^h$$

$$s.t.\quad \theta_l e_2'^{hh}(1 - b_2'^{hh}) - \theta_l e_2'^{hh}\theta_h e_1'^h b_3'^{hh} - f'^h \leq \theta_l e_2^{hl}(1 - b_2^{hl}) - \theta_l e_2^{hl}\theta_h e_1^h b_3^{hl} - f^l \quad (7.5)$$

$$\theta_h e_2'^{hh}(b_2'^{hh} + \theta_h e_1^h b_3'^{hh}) + f'^h - \frac{1}{2}(e_2'^{hh})^2 \geq \theta_h e_2^{hh}(b_2^{hh} + \theta_h e_1^h b_3^{hh}) + f^h - \frac{1}{2}(e_2^{hh})^2 \quad (7.6)$$

Assume $u = \theta_h e_2^{hh}(b_2^{hh} + \theta_h e_1^h b_3^{hh}) + f^h - \frac{1}{2}(e_2^{hh})^2$, and $\pi^{hl} = \theta_l e_1^h(1 - b_2^{hl}) - \theta_l e_2^{hl}\theta_h e_1^h b_3^{hl} - f^l$. The above program is equivalent to the following:

$$\{\beta^{hh}, b_3^{hh}, f^h\} \in arg \max_{\{\beta'^{hh}, b_3'^{hh}, f'^h\}} \theta_h^2 \beta'^{hh}(1 - \beta'^{hh}) - f'^h$$

$$s.t.\quad \frac{1}{2}\theta_h^2 \beta_h'^2 - f'^h \geq u$$

Remember in this section, we examine a contract without cross-pledging.

If the Constraint 7.5 is satisfied, the Constraint 7.6 will be satisfied too. The argument is offered below. Assume that $\{b_2^{hh}, f^h\}$ is the contract that satisfies the following program. I'll prove that the principal will not want to renegotiate this contract as long as the truth-telling constraint is satisfied.

$$\begin{aligned}\{b_2^{hh}, f^h\} \quad &\in arg \max_{\{b_2'^{hh}, f'^h\}} \theta_h e_2'^{hh}(1 - b_2'^{hh}) - f'^h \\ s.t. \quad &\theta_l e_1'^h(1 - b_2'^{hh}) - f'^h \leq \theta_l e_1^h(1 - b_2^{hl}) - f^l\end{aligned}$$

Assume $\{b_2'^{hh}, f'^h\}$ is the renegotiated contract, from which the principal obtains a higher profit than from the old contract $\{b_2^{hh}, f^h\}$. The following inequalities are met:

$$\begin{aligned}&\theta_h^2 b_2'^{hh}(1 - b_2'^{hh}) - f'^h > \theta_h^2 b_2^{hh}(1 - b_2^{hh}) - f^h \\ \Leftrightarrow &f^h - f'^h > \theta_h^2 b_2^{hh}(1 - b_2^{hh}) - \theta_h^2 b_2'^{hh}(1 - b_2'^{hh}) \\ \Leftrightarrow &f^h - f'^h > \theta_l\theta_h b_2^{hh}(1 - b_2^{hh}) - \theta_l\theta_h b_2'^{hh}(1 - b_2'^{hh}) \\ \Leftrightarrow &\theta_l\theta_h b_2'^{hh}(1 - b_2'^{hh}) - f'^h > \theta_l\theta_h b_2^{hh}(1 - b_2^{hh}) - f^h \\ \Leftrightarrow &\theta_l\theta_h b_2'^{hh}(1 - b_2'^{hh}) - f'^h > \pi^{hl}\end{aligned}$$

The above inequality conflicts with the principal's truth-reporting constraint 7.5.

2. The second step is to show whether the principal of matching quality hl renegotiates the salary to zero depends on the separating profit. It is easy to see that b_1^h does not depend on the renegotiation as it is already sunk. Define $b_1^{hh} = b_1^{hl} = b_1^h$. Because b_1^h does not depend on the renegotiation as it is already sunk, the principal sets $b_1^h = \frac{1}{2}$. It is easy to verify that without cross-pledging, $b_1^l = \frac{1}{2}$. Following the proof in Lemma 4, Program P'^{hl} can be simplified to the following program:

$$\begin{aligned}\{b_2^{hl}, f^l\} \quad &\in arg \max_{b_2'^{hl}, f'^l} \theta_l^2 b_2'^{hl}(1 - b_2'^{hl}) - f'^l \\ s.t. \quad &\frac{1}{2}\theta_l^2(b_2'^{hl})^2 - f'^l \geq u\end{aligned}$$

If $f^l > 0$, we could easily verify that $b_2^{hl} = 1$ by substituting the constraint into the objective function. This only happens if $\theta_h \geq 2\theta_l$. From Constraint 7.3, one could verify that if $b_2^{hl} = 1$, $f^l = \frac{1}{4}\theta_l(\theta_h - 2\theta_l)$.

If $\theta_h < 2\theta_l$. The principal will not set $f^l > 0$, as it is not renegotiation-proof. The principal will always substitute it with more bonus. So the bonus will be set at the highest possible level with $f^l = 0$. According to Constraint 7.3, one could find that $b_2^{hl} = \frac{1}{2}(1 + \sqrt{\frac{\theta_h}{\theta_l} - 1})$.

Q.E.D.

Corollary 2 Without the cross-pledging effect, pooling equilibrium does not survive the Intuitive Criterion.

Proof Proof by contradiction. Assume $b_2^{hl} = b_2^{hh} = b$ and $f^h = f^l = 0$. Further assume $\bar{\theta} = q\theta_h + (1-q)\theta_l$. The agent maximizes her own utility and chooses the optimal effort level:

$$e_2^* \in arg \max_{e_2} q\theta_h e_2 b_2^{hh} + (1-q)\theta_l e_2 b_2^{hl} - \frac{1}{2}e_2^2$$

$$e_2^* = \bar{\theta} b$$

The principal of type hh obtains profit $\pi = \theta_h \bar{\theta} b(1-b)$. If she deviates by paying an additional salary $f^h = \frac{1}{2}(\theta_h - \bar{\theta})\theta_h b(1-b)$, then she could obtain profit $\pi' = \theta_h^2 b(1-b)$. And $\pi' - f^h > \pi$. Q.E.D.

Lemma 5 If information is symmetric, the principal offers the following contracts in equilibrium:

- Principal with matching quality hh offers $\{b_1^{hh} = 0, b_2^{hh} = 0, b_3^{hh} = \frac{1}{\theta_h^2}, f^{hh} = 0\}$;
- Principal with matching quality hl offers $\{b_1^{hl} = 0, b_2^{hl} = 0, b_3^{hl} = \frac{1}{\theta_l \theta_h}, f^{hl} = 0\}$;
- Principal with matching quality ll offers: $\{b_1^{ll} = 0, b_2^{ll} = 0, b_3^{ll} = \frac{1}{\theta_l^2}, f^{ll} = 0\}$.

Proof I first prove an auxiliary result: a contract that contains b_1^h and b_2^{hh} offered by a high-type principal can be replicated by a contract which does not contain b_1^h and b_2^{hh} but only b_3^{hh}.

1. I first show that a contract which contains b_1^h offered by a principal of high matching quality can be replicated by a contract which does not contain b_1^h.

   The principal's maximization program P'^h under asymmetric information can be rewritten as follows:

   $$\max_{\{b\{.\},\beta\{.\}\}} \quad (\theta_h - \theta_l)(\theta_h b_1^h + q\theta_h^3 b_3^{hh}\beta^h + (1-q)\theta_h\theta_l^2 b_3^{hl}\beta^l)(1 - b_1^h - \theta_l^2 b_3^{ll}\beta^l)$$

   Set $\theta_h^2 b_3'^{hh}\beta'^h = b_1^h + \theta_h^2 b_3^{hh}\beta^h$, and $\theta_l^2 b_3'^{hl}\beta'^l = b_1^h + \theta_l^2 b_3^{hl}\beta^l$. With $\{b_3'^{hh}, \beta'^h, b_3'^{hl}, \beta'^l\}$, the firm achieves the same profit. The agent will exert the same amount of effort e_1^h, but effort e_2 will increase due to an increase in β if b_3^{hh} and b_3^{hh} are kept constant. The principal could obtain the same profit using contract $\{b_3^{hh}, \beta'^h, b_3^{hl}, \beta'^l\}$ which induces a higher level of effort. This means that the principal could pay the agent less (less rent to the agent) in order to obtain a higher profit.

   The principal could use higher b_2 to keep the first period effort because of the cross-pledging effect while increasing the second period effort.

2. I then show a contract that contains b_2^{hh} offered by a principal of high matching quality can be replicated by a contract which does not contain b_2^{hh}.

   b_3^{hh} enters into the maximization program through the term $(\theta_h - \theta_l)q\theta_h^3 b_3^{hh}\beta^h(1 - b_1^h - \theta_l^2 b_3^{ll}\beta^l)$. One could show that the principal will always want to use b_3^{hh} to substitute b_2^{hh}. Assume that $b_2'^{hh} = b_2^{hh} - \epsilon$, and $\theta_h e_1^h b_3'^{hh} = \theta_h e_1^h b_3'^{hh} + \epsilon$. The second equation implies $b_3'^{hh} > b_3^{hh}$, and $\beta'^h = \beta^h$ if e_1^h is not affected. However, $e_1^h = \theta_h b_1 + \theta_h^3\beta^h b_3^{hh} = \theta_h^3\beta^h b_3^{hh}$. When b_3^{hh} goes up to $b_3'^{hh}$, $e_1'^h > e_1'^h$ and $\beta'^h > \beta^h$, leading to a higher profit.

   The principal uses b_3^{hh} instead of b_2^{hh} as the former also induces higher first period effort because of the cross-pledging effect.

I then prove Lemma 5.

1. Under contract $\{b\{.\}, f\{.\}\}$, the agent of constantly low matching quality could obtain

utility level following Lemma 4:

$$
\begin{aligned}
&\theta_l b_1^{ll} e_1^l + \theta_l e_2^{ll} b_2^{ll} + \theta_l e_1^l \theta_l e_2^{ll} b_3^{ll} - \frac{1}{2}(e_1^l)^2 - \frac{1}{2}(e_2^{ll})^2 \\
\Leftrightarrow & e_1^l(\theta_l b_1^{ll} - \frac{1}{2}(e_1^l)) + \frac{1}{2}\theta_l^2 \beta^{ll} \\
\Leftrightarrow & \frac{1}{2}(\theta_l b_1^l + \theta_l^3 b_3^{ll} \beta^{ll})(\theta_l b_1^l - \theta_l^3 b_3^{ll} \beta^{ll}) + \frac{1}{2}\theta_l^2 \beta^{ll} \\
\Leftrightarrow & \frac{1}{2}(\theta_l^2 (b_1^l)^2 - \theta_l^6 (b_3^{ll})^2 (\beta^{ll})^2) + \frac{1}{2}\theta_l^2 \beta^{ll}
\end{aligned}
$$

One could find a contract which consists of only b_3^{ll} to incentivise the agent. $e_2^{ll} = \theta_l^2 e_1^l b_3^{ll}$. As a result, the agent's utility

$$
\begin{aligned}
&\frac{1}{2}\theta_l^4 (e_1^l)^2 (b_3^{ll})^2 - \frac{1}{2}(e_1^l)^2 \\
\Leftrightarrow & \frac{1}{2}(e_1^l)^2 (\theta_l^4 (b_3^{ll})^2 - 1)
\end{aligned}
$$

b_3^{ll} is set at such a level that the following equation is satisfied:

$$
\frac{1}{2}(e_1^l)^2 (\theta_l^4 (b_3^{ll})^2 - 1) = \frac{1}{2}(\theta_l^2 (b_1^l)^2 - \theta_l^6 (b_3^{ll})^2 (\beta^{ll})^2) + \frac{1}{2}\theta_l^2 \beta^{ll}
$$

As a result, $b_3^{ll} = \frac{1}{\theta_l^2}$, $e_1^l = e_2^{ll} = 1$.

2. Following the same argument in the previous step, principal of high matching quality at date 0 only uses b_3^{hl} and b_3^{hh}. The agent's expected utility is as follows:

$$
\frac{1}{2}(e_1^h)^2 \{q(\theta_h^4 (b_3^{hh})^2 - 1) + (1-q)(\theta_h^2 \theta_l^2 (b_3^{hl})^2 - 1)\}
$$

The minimum compensation paid to the agent in order to induce effort level 1 is by setting $q(\theta_h^4 (b_3^{hh})^2 - 1) + (1-q)(\theta_h^2 \theta_l^2 (b_3^{hl})^2 - 1) = 0$. The principal's maximization problem is:

$$
\begin{aligned}
\max_{\{b_3^{hl}, b_3^{hh}\}} \; & q(\theta_h e_1^h + \theta_h e_2^{hh} - \theta_h e_1^h \theta_h e_2^{hh} b_3^{hh}) + (1-q)(\theta_h e_1^h + \theta_l e_2^{hl} - \theta_h e_1^h \theta_l e_2^{hl} b_3^{hl}) \\
s.t. \; & q(\theta_h^4 (b_3^{hh})^2 - 1) + (1-q)(\theta_h^2 \theta_l^2 (b_3^{hl})^2 - 1) = 0
\end{aligned}
$$

Because $e_2^{hl} = \theta_l \theta_h e_1^h b_3^{hl}$ and $e_2^{hh} = \theta_h^2 e_1^h b_3^{hh}$, it is equivalent to the following program:

$$\max_{\{e_2^{hh}, e_2^{hl}\}} q\theta_h e_2^{hh} + (1-q)\theta_l e_2^{hl}$$

$$s.t. \quad q(e_2^{hh})^2 + (1-q)(e_2^{hl})^2 = 1$$

Because $e_2^{hh}, e_2^{hl} \leq 1$, the principal set$e_2^{hh}, e_2^{hl} = 1$ to maximize the profit. Thus $b_3^{hl} = \frac{1}{\theta_h \theta_l}$, and $b_3^{hh} = \frac{1}{\theta_h^2}$. $\beta^{hh} = \frac{1}{\theta_h}$, $\beta^{hl} = \frac{1}{\theta_l}$ and $\beta^{ll} = \frac{1}{\theta_l}$.

When the two parties receive new information in period two, the agent will not want to renegotiate. The agent's utility of constant high matching quality is $\theta_h e_2^{hh} \beta^{hh} - \frac{1}{2}(e_2^{hh})^2 = \frac{1}{2}\theta_h^2(\beta^{hh})^2$. Under the contract analyzed above, the agent's utility is $\frac{1}{2}$. If the principal wants to renegotiate and sets $\beta^{hh} = \frac{1}{2}$, the agent's utility would be $\frac{1}{8}\theta_h^2$. The agent thus will not want to renegotiate. Q.E.D.

Proposition 4

- The principal of high matching quality commits to the following contract in the first period: $\{b_1^{hh} = 0, b_2^{hh} = 0, b_3^{hh} = \sqrt{\frac{1-(1-q)\theta_l^2}{q\theta_h^4}}, f^{hh} = f^{hl} + \theta_l \theta_h^2 b_3^{hh}(1 - \theta_h b_3^{hh}); b_1^{hl} = 0, b_2^{hl} = \frac{3q\theta_l^2 - 2q^2\theta_l^2 - q\theta_h + q^2\theta_h - \theta_l^2 + 1}{2q(1-q)\theta_l^2}, b_3^{hl} = \frac{1}{2\theta_l^2} - \frac{\theta_h b_3^{hh}}{2(1-q)\theta_l^2}, f^{hl} = 1 - \theta_l + (\theta_h - \theta_l)\theta_l^2 b_3^{hl}\}$.
- The principal of low matching quality offers the following contract in the first period: $\{b_1^{ll} = 0, b_2^{ll} = 0, b_3^{ll} = \frac{1}{\theta_l^2}, f^{ll} = 0\}$.

Proof The principal's maximization program P'^h under asymmetric information can be rewritten as follows:

$$\max_{\{b\{.\},\beta\{.\}\}} \quad (\theta_h - \theta_l)(\theta_h b_1^h + q\theta_h^3 b_3^{hh}\beta^{hh} + (1-q)\theta_h\theta_l^2 b_3^{hl}\beta^{hl})(1 - b_1^h - \theta_l^2 b_3^{ll}\beta^{hl})$$

Take first order derivative w.r.t. b_3^{hl}, one could find that:

$$b_3^{hl} = \frac{1}{2\theta_l^2}(1 - b_1^h) - \frac{b_1^h + \theta_h^2 \beta^{hh} b_3^{hh}}{2(1-q)\theta_l^2}$$

In the second stage renegotiation for a principal of type hl, program P'^{hl} is as follows:

$$\begin{aligned}\{\beta^{hl}, b_3^{hl}, f^l\} \quad &\in arg \max_{\{\beta'^{hl}, b_3'^{hl}, f'^l\}} \theta_l^2 \beta'^{hl}(1-\beta'^{hl}) - f'^l \\ s.t. \quad & \frac{1}{2}\theta_l^2 \beta'^{hl2} - f'^l \geq u\end{aligned}$$

If $f^l > 0$, then $\beta^{hl} = 1$. The principal of constant high matching quality will not want to renegotiate the contract as proved in Proposition 3. As a result,

$$b_3^{hl} = \frac{1}{2\theta_l^2} - \frac{\theta_h^2 \beta^{hh} b_3^{hh}}{2(1-q)\theta_l^2}$$

The agent hired by principal of high matching quality at date 0 chooses effort level e_1^h:

$$\begin{aligned}e_1^h \quad &\in arg \max_{e_1^h} q(\theta_h e_1^h \theta_h e_2^{hh} b_3^{hh} - \frac{1}{2}(e_2^{hh})^2) + (1-q)(\theta_l e_2^{hl}\beta^{hl} - \frac{1}{2}(e_2^{hl})^2) - \frac{1}{2}(e_1^h)^2 \\ \Leftrightarrow &\in arg \max_{e_1^h} \frac{1}{2} q(\theta_h^4 (b_3^{hh})^2 - 1)(e_1^h)^2 + \frac{1}{2}(1-q)(\theta_l^2(\beta^{hl})^2 - 1)(e_1^h)^2\end{aligned}$$

To induce the agent to make an effort $e_1^h = 1$, the principal sets b_3^{hh} at:

$$\begin{aligned}b_3^{hh} &= \sqrt{\frac{1-(1-q)\theta_l^2(\beta^{hl})^2}{q\theta_h^4}} \\ &= \sqrt{\frac{1-(1-q)\theta_l^2}{q\theta_h^4}}\end{aligned}$$

It can be easily verified that $b_3^{hh} > \frac{1}{\theta_h^2}$. As a result, $b_3^{hl} < 0$. The principal sets b_2^{hl} at:

$$\begin{aligned}b_2^{hl} &= 1 - \theta_h b_3^{hl} \\ &= 1 - \theta_h (\frac{1}{2\theta_l^2} - \frac{\theta_h^2 \beta^{hh}(b_3^{hh})^2}{2(1-q)\theta_l^2}) \\ &= \frac{q\theta_h - q^2\theta_h + \theta_l^2 - q\theta_l^2 - 1}{2q(1-q)\theta_l^2} \\ &= \frac{3q\theta_l^2 - 2q^2\theta_l^2 - q\theta_h + q^2\theta_h - \theta_l^2 + 1}{2q(1-q)\theta_l^2}\end{aligned}$$

Because $b_3^{hl} < 0$, if the principal has no limited liability, then $b_2^{hl} > 1$.

$$
\begin{aligned}
f^l &= \theta_l + \theta_l^2(\theta_h - \theta_l)b_3^{hl} - (2\theta_l - 1) \\
&= 1 - \theta_l + (\theta_h - \theta_l)\theta_l^2 b_3^{hl} \\
f^h &= f^l + \theta_l - \theta_l\theta_h b_3^{hh} - \theta_l(1 - b_2^{hl}) + \theta_l\theta_h e_2^{hl} b_3^{hl} \\
&= f^l + \theta_l\theta_h^2 b_3^{hh}(1 - \theta_h b_3^{hh})
\end{aligned}
$$

b_3^{hh} is decreasing in q. f^l is increasing in $\theta_h - \theta_l$. Q.E.D.

Lemma 6 With the cross-pledging effect, pooling equilibrium does not survive Intuitive Criterion.

Proof Proof by contradiction. Assume $b_2^{hl} = b_2^{hh} = b_2$, $b_3^{hl} = b_3^{hh} = b_3$ and $f^h = f^l = 0$. Further assume $\bar{\theta} = q\theta_h + (1-q)\theta_l$. The agent maximizes her own utility and chooses the optimal effort level:

$$
\begin{aligned}
e_2^* \quad &\in arg\max_{e_2} q(\theta_h e_1 \theta_h e_2 b_3 + \theta_h e_2 b_2) + (1-q)(\theta_h e_1 \theta_l e_2 b_3 + \theta_l e_2 b_2) - \frac{1}{2}e_2^2 \\
e_2^* &= \theta_h \bar{\theta} e_1 b_3 + \bar{\theta} b_2
\end{aligned}
$$

The principal of type hh obtains profit $\pi = \theta_h e_2^*(1 - b_2 - \theta_h e_1 b_3)$. If she deviates by paying an additional salary $f^h = \frac{1}{2}(\theta_h - \bar{\theta})(\theta_h e_1 b_3 + b_2)(1 - b_2 - \theta_h e_1 b_3)$, then she could obtain profit $\pi' = \theta_h e_2'(1 - b_2 - \theta_h e_1 b_3)$, in which $e_2' = \theta_h^2 e_1 b_3 + \theta_h b_2$. And $\pi' - f^h > \pi$. Q.E.D.